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Fractions as percepts? Exploring cross-format distance effects for fractional magnitudes

Percival G. Matthews [a,*], Dana L. Chesney [b]

[a] *Department of Educational Psychology, University of Wisconsin-Madison, 1025 W. Johnson Street, #884, Madison, WI 53706-1796, USA*
[b] *Department of Psychology, The Ohio State University, 225 Psychology Building, 1835 Neil Avenue, Columbus, OH 43210, USA*

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ABSTRACT

This study presents evidence that humans have intuitive, perceptually based access to the abstract fraction magnitudes instantiated by nonsymbolic ratio stimuli. Moreover, it shows these perceptually accessed magnitudes can be easily compared with symbolically represented fractions. In cross-format comparisons, participants picked the larger of two ratios. Ratios were presented either symbolically as fractions or nonsymbolically as paired dot arrays or as paired circles. Response patterns were consistent with participants comparing specific analog fractional magnitudes independently of the particular formats in which they were presented. These results pose a challenge to accounts that argue human cognitive architecture is ill-suited for processing fractions. Instead, it seems that humans can process nonsymbolic ratio magnitudes via perceptual routes and without recourse to conscious symbolic algorithms, analogous to the processing of whole number magnitudes. These findings have important implications for theories regarding the nature of human number sense – they imply that fractions may in some sense be natural numbers, too.

1. Introduction

Formal number concepts and the mathematics built upon them were invented too recently to have influenced the evolution of our species (Dehaene & Cohen, 2007). How is it then that evolutionarily

* Corresponding author. Fax: + 608 262 0843.
E-mail addresses: pmatthews@wisc.edu (P.G. Matthews), dlchesney@gmail.com (D.L. Chesney).

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ancient human brains can support these relatively recent numerical inventions? To address this question, researchers often look to the basic cognitive architectures upon which culturally established number concepts might be built. The counting numbers (i.e., 1, 2, 3...) – which mathematicians have dubbed 'natural' numbers – are often the focal point of these theories. It makes intuitive sense that these 'natural' numbers might form the groundwork of our understanding of mathematics. These numbers play a major role not just in counting, but in numerical cognition more generally (Butterworth, 2010; Gerstmann, 1940; Noël, 2005). Moreover, they map onto basic perceptual abilities that enumerate discrete sets. This ability to perceptually estimate discrete numerical magnitudes – an ability granted by what is known as the approximate number system (ANS) – is present not only in humans but across multiple species (e.g., Dehaene, Dehaene-Lambertz, & Cohen, 1998; Meck & Church, 1983). Indeed, several researchers have argued that the acquisition of abstract numerical concepts rests upon these evolutionarily inherited enumeration abilities (e.g., Dehaene, 1997; Feigenson, Dehaene, & Spelke, 2004; Nieder, 2005; Piazza, 2010). By positing such a crucial role for perceptually based enumeration in the development of number concepts, these theories privilege natural numbers by proxy, essentially echoing Kroenecker's famous dictum that "God made the integers; all the rest is the work of man" (Bell, 1986, p. 477).

However, this ostensibly obvious intuition may obscure the possibility that natural numbers and enumeration are not alone in their 'naturalness'. In this study, we administered cross-format comparison tasks to explore whether humans have an intuitive sense of nonsymbolic ratio magnitude that allows them to perceive and judge fractional[1] number values in ways similar to how the approximate number system allows them to perceive and judge natural number magnitudes. The cross format nature of the comparisons is important: Successful comparison within a particular format might be accomplished by methods that need not necessarily require magnitude abstraction, such as scaling (e.g., Ahl, Moore, & Dixon, 1992).

By contrast, cross-format comparisons require some sort of abstraction of magnitudes to allow comparison on the same scale. Each comparison involved fraction magnitudes instantiated in nonsymbolic forms that were not amenable to simple enumeration or to manipulation via symbolic algorithms, insuring any such abstractions must be perceptually based.

Minimally, two pieces of evidence seem important to support the possibility that participants perceive abstract ratio magnitudes:

1. Participants must prove sensitive to the equivalence of fraction values across formats when perception is the only plausible route to identifying those magnitudes, and
2. Participants must complete comparisons in a short enough time course to preclude the use of conscious algorithms.

Patterns consistent with these constraints would suggest that this sense of proportion is unlikely to be dependent upon enumeration or estimation of natural number values. In short, such results would imply that fractions are in some sense 'natural' numbers too.

1.1. Primitive ratio processing – A link to natural fraction concepts?

The proposition that fractional number values may be intuitive might seem at odds with the fact that both children and even highly educated adults often experience considerable difficulties understanding symbolic fractions (e.g., Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981; Ni & Zhou, 2005; Siegler & Pyke, 2012). For instance, when a nationally representative sample of children was asked whether 12/13 + 7/8 was closest to 1, 2, 19, or 21, 8th-graders chose 19 and 21 more often than 2 (Carpenter et al., 1981). These problems extend well past middle school, persisting into adulthood. On the same estimation problem, a nationally representative sample of 17-yr-olds was correct only 37% of the time. Moreover, Stigler, Givvin, and Thompson (2010) found that only 33% of their sample of community college students could accurately find the largest of four simple fractions. Many have

[1] Although we note that a mathematically rigorous treatment of the terms 'fraction' and 'ratio' considers ratio to be one of several possible interpretations of fraction concepts, we will use the terms interchangeably throughout this manuscript.