Cognitive Psychology 74 (2014) 1–34

Contents lists available at ScienceDirect

Cognitive Psychology

journal homepage: www.elsevier.com/locate/cogpsych

Extending problem-solving procedures through reflection ☆

John R. Anderson *, Jon M. Fincham

Department of Psychology, Carnegie Mellon University, 5000 Forbes Ave., Pittsburgh, PA 15213, USA

A R T I C L E I N F O

Article history:
Accepted 18 June 2014
Available online 24 July 2014

Keywords:
Mathematical problem solving
Reflection
Cognitive modeling
Multivoxel pattern analysis
Hidden Markov models

A B S T R A C T

A large-sample ($n = 75$) fMRI study guided the development of a theory of how people extend their problem-solving procedures by reflecting on them. Both children and adults were trained on a new mathematical procedure and then were challenged with novel problems that required them to change and extend their procedure to solve these problems. The fMRI data were analyzed using a combination of hidden Markov models (HMMs) and multi-voxel pattern analysis (MVPA). This HMM–MVPA analysis revealed the existence of 4 stages: Encoding, Planning, Solving, and Responding. Using this analysis as a guide, an ACT-R model was developed that improved the performance of the HMM–MVPA and explained the variation in the durations of the stages across 128 different problems. The model assumes that participants can reflect on declarative representations of the steps of their problem-solving procedures. A Metacognitive module can hold these steps, modify them, create new declarative steps, and rehearse them. The Metacognitive module is associated with activity in the rostrolateral prefrontal cortex (RLPFC). The ACT-R model predicts the activity in the RLPFC and other regions associated with its other cognitive modules (e.g., vision, retrieval). Differences between children and adults seemed related to differences in background knowledge and computational fluency, but not to the differences in their capability to modify procedures.

☆ The analyses and models in this paper can be obtained at http://act-r.psy.cmu.edu/?post_type=publications&p=16145.

* Corresponding author.
E-mail addresses: ja@cmu.edu (J.R. Anderson), fincham@cmu.edu (J.M. Fincham).

http://dx.doi.org/10.1016/j.cogpsych.2014.06.002

1. Introduction

While some instruction has as its goal that the learner become skilled at just what is being taught, in many cases the goal is for the learner to be able to transfer what is learned to new situations. The literature abounds with demonstrations of both failed transfer (e.g., Bassok, 1990; Detterman, 1993; Gick & Holyoak, 1980) and near total transfer (e.g., Bovair et al., 1980; Singley & Anderson, 1989). Educators properly anguish over the implications of these apparently contradictory results (e.g., Bransford & Schwartz, 1999; Carraher & Schliemann, 2002).

One of the reasons for the different perspectives on transfer is the wide variety of things that can transfer. They can range from transfer of highly proceduralized skills such as from one kind of manual transmission to another to what might better be called discovery such as the connection made between the structure of the solar system and the structure of the atom. This paper will focus on a particular type of transfer – where one derives new solution procedures by extending problem-solving procedures that one already knows. It is particularly important in mathematics learning, which is the content focus of this paper. To take a modest example, children who learn the basic principles for solving equations need to apply them successfully to an infinite space of equations. To take a more ambitious example, mathematics education hopes that students will transfer what they learn in the classroom to being successful workers and informed citizens.

More specifically, this paper will consider situations where participants need to reflect on a known procedure and modify and replace parts of it. For instance, people often face such a situation when a favorite piece of software is upgraded. It is an explicit goal of the National Council of Teachers of Mathematics (NCTM) standards (Romberg, 1992) that students should be able to "generate new procedures and extend or modify familiar ones."

This paper will develop a theory of procedural extension within the ACT-R theory (Anderson, 2007; Anderson et al., 2004; Salvucci, 2013; Taatgen, Huss, Dickison, & Anderson, 2008) of procedure following. The ACT-R theory holds that both verbal procedural instructions and examples of procedures are initially encoded as declarative representations of problem-solving steps, which are retrieved and interpreted in solving a problem. Note that declarative encodings of procedures are not the sort of unconscious "procedures" that occupy much of the discussion about the procedural–declarative distinction in psychology (e.g., Cohen, Poldrack, & Eichenbaum, 1997; Willingham, Nissen, & Bullemer, 1989). With enough practice such declarative knowledge can be compiled into production rules in ACT-R, which are one form of unconscious procedures.

Recently, Taatgen (2013) has produced an ACT-R theory of transfer in which steps from one procedure automatically transfer to another procedure. This is not the reflective transfer considered here. This paper is concerned with situations where one consciously reflects on what one knows and how to extend that knowledge. A classic example would be Wertheimer's (1945/1959) study of how children could use what they know of the area of rectangles to find the area of a parallelogram.

2. ACT-R, procedure following, and fMRI

As background for the current research, we will briefly review the ACT-R theory, how procedure following is modeled, and how the activity of components in the ACT-R theory have been related to fMRI measures. ACT-R 6.0 (Anderson, 2007) consists of a set of different modules whose interactions are controlled by a production system. Different modules are specialized to achieve specific goals. Of relevance to this paper, the Manual module programs the hands, the Visual module encodes visual input, the Retrieval module accesses declarative information, and the Imaginal module manipulates mental representations. These modules put products into module-specific buffers – for instance, representation of a visual object into the Visual buffer or a retrieved memory into a Retrieval buffer. Productions can detect information in these buffers and make requests of modules. For instance, a production can detect an object in the Visual buffer and request the Retrieval module find declarative information relevant to that object.

The ACT-R theory of declarative procedure following has had considerable success in modeling the learning of a number of procedures including simple algebra (e.g., Anderson, 2005). According to