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Understanding decimal proportions: Discrete representations, parallel access, and privileged processing of zero

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ABSTRACT

Much of the research on mathematical cognition has focused on the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9, with considerably less attention paid to more abstract number classes. The current research investigated how people understand decimal proportions – rational numbers between 0 and 1 expressed in the place-value symbol system. The results demonstrate that proportions are represented as discrete structures and processed in parallel. There was a *semantic interference effect*: When understanding a proportion expression (e.g., "0.29"), both the correct proportion referent (e.g., *0.29*) and the incorrect natural number referent (e.g., *29*) corresponding to the visually similar natural number expression (e.g., "29") are accessed in parallel, and when these referents lead to conflicting judgments, performance slows. There was also a *syntactic interference effect*, generalizing the unit-decade compatibility effect for natural numbers: When comparing two proportions, their tenths and hundredths components are processed in parallel, and when the different components lead to conflicting judgments, performance slows. The results also reveal that zero decimals – proportions ending in zero – serve multiple cognitive functions, including eliminating semantic interference and speeding processing. The current research also extends the distance, semantic congruence, and SNARC effects from natural numbers to decimal proportions. These findings inform how people understand the place-value symbol system, and the mental implementation of mathematical symbol systems more generally.

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1. Introduction

Much of the research in mathematical cognition has focused on how people understand the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9. The behavioral evidence suggests that adults and children represent these numbers as magnitudes and process them psychophysically (Moyer & Landauer, 1967; Sekuler & Mierkiewicz, 1977). The neuroscience evidence suggests that the intra-parietal sulcus is the primary neural correlate of these magnitude representations (Cantlon, Brannon, Carter, & Pelphrey, 2006; Pinel, Dehaene, Rivière, & Le Bihan, 2001). These findings matter: The developmental and educational evidence suggests that weakened magnitude representations are associated with lower mathematical achievement, both at the low end of the normal range (De Smedt, Verschaffel, & Ghesquière, 2009; Halberda, Mazzocco, & Feigenson, 2008) and in dyscalculia (Butterworth, Varma, & Laurillard, 2011; Mazzocco, Feigenson, & Halberda, 2011; Piazza et al., 2010).

Although research on the numbers 1–9 has bootstrapped mathematical cognition, the time has come for systematic investigation of more abstract number classes, which have thus far received less attention in the literature (Siegler, Thompson, & Schneider, 2011; Varma & Schwartz, 2011). These include the natural numbers, which extend the numbers 1–9 with zero and with positive multi-digit whole numbers; the integers, which extend the natural numbers with negative whole numbers; the rationals, which extend the integers with ratios of integers; and so on. The current research focuses on *decimal numbers*, which are rational numbers expressed in the place-value symbol system with base 10 (hereafter the *decimal symbol system*). Investigating how people understand the decimal symbol system provides insight into how they understand mathematical symbol systems more generally.

We consider three research questions. The first concerns the *semantics* of decimal numbers. The place-value symbol system is extraordinarily efficient, using a small set of symbols to name a large set of numbers. With this efficiency comes the potential for interference. For example, the natural number expression "29" is visually similar to the integer expression "−29" and the proportion expression "0.29", even though their referents are quite different. Can people selectively access the correct referent of a number expression? Or do they suffer interference from incorrect referents named by visually similar number expressions?

The second research question concerns the *syntax* of decimal numbers. Do decimal numbers have structured representations that reify the syntax of the place-value symbol system? Or are they represented continuously, as extensions of the magnitude representations of the numbers 1–9?

The third research question concerns zero, the *null symbol* of the place-value symbol system. The mathematical function of zero – to indicate that a place has no value – is critical for expressing the difference between 0.209 and 0.290, for example. We consider two hypotheses regarding its cognitive function. The first is that zero is used in decimal expressions to eliminate semantic ambiguity. The second is that *zero decimals* – natural numbers and proportions ending in zero – are privileged in cognitive processing because of their frequency and cultural importance.

1.1. Terminology

Before beginning, we introduce terminology for describing decimal numbers. Lexically, *decimal expressions* are sequences of digits ("1", "2", ... , "9") and up to one decimal point ("."), optionally prefixed by a negative sign ("−").

Semantically, decimal expressions denote rational numbers, which we refer to as *decimal referents*. For example, the decimal expression "29" has the decimal referent *29*. (Decimal expressions are indicated with quotes and decimal referents with italics, and plain text is used when this distinction is unimportant.) Semantics is critical: Although the decimal expressions "29" and "0.29" are visually similar, their decimal referents differ by two orders of magnitude.

Syntactically, the structure of decimal expressions is defined by the decimal symbol system. For example, the decimal expression "29" has a tens place with value "2" and a ones place with value "9". Syntax is also critical: Although "29", "92", and "0.29" are visually similar, they are different decimal expressions denoting different decimal referents.