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ScienceDirect
Forest Policy and Economics 9 (2007) 1031 – 1045
Forest Policy and Economics
www.elsevier.com/locate/forpol

Determination of optimal rotation period under stochastic wood and carbon prices

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Received 30 September 2005; received in revised form 10 September 2006; accepted 19 September 2006

Abstract

The valuation of forest stands is traditionally based on a profit calculus involving revenue from wood sales and associated costs. Currently, the role of carbon management in forests is actively discussed. In a stochastic setting we extend the analysis of the optimal rotation period by considering uncertain revenue streams from carbon trading. We develop a real options model given uncertainties in future wood and CO_2 price behaviour. A detailed sensitivity analysis of the numerical results for both cases – with and without carbon sequestration – is provided. We find that optimal rotation periods vary considerably with (i) the type of price process, (ii) the way how carbon income is defined, and (iii) the selection of discount rates.

Keywords: Optimal rotation; Carbon sequestration; Real options

1. Introduction

The controversy of optimal forest management has a long history in forestry economics. The proposal, in 1849, by Faustmann to consider discounting in a dynamic setting has substantially impacted the thinking about optimal forest management (Faustmann, 1849). In particular, in central Europe the Faustmann model has been raising debates since its inception and discussions on a more international level continue to be lively (see e.g. Brazee, 2001; Newman, 2002).

During the last three decades many authors attempted to extend the tree-cutting problem, in order to incorporate stochastic behaviour. The first studies within such context has probably been papers by Lembersky and Johnson (1975) and Brock et al. (1982). In Lembersky and Johnson (1975) authors deal with the stand management problem under price and stand response uncertainty in infinite time horizon. The uncertainties are modelled using state transition probabilities in discrete space. Since Lembersky and Johnson consider only a finite number of states and management actions, the optimal policy can be expressed as a solution

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doi:10.1016/j.forpol.2006.09.005

to a specific functional equation and thus obtained iteratively. On the other hand, Brock et al. consider the harvesting problem in a more general context: the context of stochastic capital theory. They model the tree size using both discrete and continuous processes and prove that the so-called *barrier strategy* is optimal. However, their model setup is only a simple one: the value of the tree they consider is given merely by its size. In a subsequent paper Miller and Voltaire (1983) extended the work by Brock et al. (1982) to the analysis of the ongoing rotation problem.

As Newman (2002) summarizes, since the late 1980s researchers' interest in the optimal harvesting problem under uncertainty has been increasing rapidly. The stylized problems considering different type of uncertainties have been developed in an attempt to extend the Faustmann's formula for ongoing rotations on one hand and the so-called Wicksellian tree-cutting problem (that is, the single rotation setup) on the other hand, in order to handle stochastic behaviour. For example, the papers by Brazee and Mendelsohn (1988), Haight and Holmes (1991), and Clarke and Reed (1989), belong to the most significant contributions to the field of the optimal harvesting problem and were a foundation for many other publications that appeared in the last decade.

In particular, Brazee and Mendelsohn analyze an impact of price uncertainty within Faustmann's framework, assuming that clear-cut is the only management decision. Haight and Holmes pointed out the importance of the specification of the underlying timber price model. Using the dynamic programming formulation in discrete space they have determined the effect of different empirically tested timber price models on the optimal harvest policy. Clarke and Reed address the optimal harvesting problem in the continuous-time. Assuming the price and growth uncertainty (both modelled by a geometric Brownian motion) they have derived the harvesting rules for single rotations and expressed the optimal cutting rule as a barrier.

Even in the first studies on the optimal rotation under uncertainty (cf. Brock et al., 1982) many authors have pointed out that the problem of determining the single optimal rotation (or, equally, the Wicksellian tree-cutting problem) is actually a sort of optimal stopping problem known from various economic areas. In corporate finance theory the capital budgeting problems of this type (i.e. the problems related to uncertainty for which it is important to determine management's strategic reaction to the changing conditions) are nowadays commonly analyzed within the real options framework (for more details see, e.g., Dixit and Pindyck, 1994). The papers by Morck et al. (1989) and Thomson (1992) are probably among the first applications in the forestry, which have adopted the real options approach and employed the solution techniques known from the financial options theory. Thomson has proposed the model in a discrete framework and applied the lattice method. Morck et al. on the other hand have calculated the value of forestry lease under stochastic timber price and stochastic timber inventory using the contingent claim approach (cf. Merton, 1973).

More recently, a number of studies applying the real options methodology to forest management problems have been published. Their application areas differ mostly in the model formulation, in particular the way how uncertainty is treated. Commonly, the wood price or forest growth, or even both, are assumed to be stochastic (Insley, 2002; Saphores, 2003; Alvarez and Koskela, 2004b). In addition, Alvarez and Koskela (2004a) have analyzed optimal forest management under stochastic interest rate. Nonetheless, these studies provide different types of results: either a numerical or theoretical analysis of the problem is presented. In particular, Insley concentrates on the numerical illustration (when the tree harvesting problem is formulated as a linear complementarity problem), while Saphores, Alvarez and Koskela present mainly the theoretical deductions characterizing the optimal harvesting policy. The question of the proper specification of the underlying uncertainty has also been addressed: the timber price development is either described by the geometric Brownian motion or a mean-reverting process, the latter being more suitable for analyzing empirical data (see Insley, 2002).

Except the real options applications there appeared other approaches related to the stochastic optimal rotation models. For instance, Willassen (1998) applied impulse control method to the problem of generalization of the Faustmann's formula to the case of stochastic forest growth, and Lu and Gong (2003) determined the optimal thinning and harvest strategy by optimizing the coefficient of the optimal stocking level function. Despite this fact, we believe that the shift from the Faustmann's traditional net present value (NPV) approach to the real options framework has been a major innovation