Journal of Experimental Child Psychology 140 (2015) 211–227

Contents lists available at ScienceDirect

Journal of Experimental Child Psychology

journal homepage: www.elsevier.com/locate/jecp

Individual differences in algebraic cognition: Relation to the approximate number and semantic memory systems

David C. Geary *, Mary K. Hoard, Lara Nugent, Jeffrey N. Rouder

Department of Psychological Sciences, University of Missouri, Columbia, MO 65211, USA

ARTICLE INFO

Article history:
Received 25 May 2015
Revised 14 July 2015
Available online 7 August 2015

Keywords:
Algebra
Approximate number system
Hippocampal memory system
Symbolic number
Fact retrieval
Individual differences
Mathematical cognition
Achievement

ABSTRACT

The relation between performance on measures of algebraic cognition and acuity of the approximate number system (ANS) and memory for addition facts was assessed for 171 ninth graders (92 girls) while controlling for parental education, sex, reading achievement, speed of numeral processing, fluency of symbolic number processing, intelligence, and the central executive component of working memory. The algebraic tasks assessed accuracy in placing *x,y* pairs in the coordinate plane, speed and accuracy of expression evaluation, and schema memory for algebra equations. ANS acuity was related to accuracy of placements in the coordinate plane and expression evaluation but not to schema memory. Frequency of fact retrieval errors was related to schema memory but not to coordinate plane or expression evaluation accuracy. The results suggest that the ANS may contribute to or be influenced by spatial–numerical and numerical-only quantity judgments in algebraic contexts, whereas difficulties in committing addition facts to long-term memory may presage slow formation of memories for the basic structure of algebra equations. More generally, the results suggest that different brain and cognitive systems are engaged during the learning of different components of algebraic competence while controlling for demographic and domain general abilities.

* Corresponding author. Fax: +1 573 882 7710.
E-mail address: gearyd@missouri.edu (D.C. Geary).

http://dx.doi.org/10.1016/j.jecp.2015.07.010
0022-0965/

Introduction

Competence with algebra is the foundation for learning the more complex mathematics that is demanded in science, technology, engineering, and mathematical (STEM) fields (National Mathematics Advisory Panel, 2008) and contributes to employability and wages in many blue-collar occupations (Bynner, 1997). Thus, it is not surprising that improving students' learning of algebra is an educational priority, but this learning has proven to be difficult to achieve for many students (Stein, Kaufman, Sherman, & Hillen, 2011). Efforts to improve algebraic learning have included the identification of poor prerequisite knowledge, such as competence with fractions, to serve as a focus of remedial efforts (Fuchs et al., 2013; Siegler et al., 2012) as well as the development of different instructional approaches for students with different levels of algebraic expertise (Rittle-Johnson & Star, 2009; Rittle-Johnson, Star, & Durkin, 2009).

The latter may be influenced by individual differences in the more basic cognitive systems that support algebraic learning directly or the learning of prerequisite skills, including arithmetic. Intervening with these systems may provide a useful adjunct to interventions that focus on specific algebraic content (see Park & Brannon, 2013, 2014). We have taken a first step in this direction by examining the relation between specific aspects of algebraic cognition and basic cognitive systems that are correlated with individual differences in arithmetic learning, specifically the approximate number system (ANS) and the semantic memory system involved in learning basic arithmetic facts (De Smedt, Holloway, & Ansari, 2011; Geary, Hoard, & Bailey, 2012; Halberda, Mazzocco, & Feigenson, 2008; Mazzocco, Feigenson, & Halberda, 2011a, 2011b; Qin et al., 2014).

ANS and memory system

The ANS is an inherent system for representing, comparing, and combining the magnitudes of collections of objects (see Feigenson, Dehaene, & Spelke, 2004; Geary, Berch, & Mann Koepke, 2015), and there is some evidence that poor acuity of this system contributes to difficulties in learning mathematics (Piazza et al., 2010) and to individual differences in mathematics achievement more generally (Chen & Li, 2014; Fazio, Bailey, Thompson, & Siegler, 2014; Kibbe & Feigenson, 2015; Libertus, Halberda, & Feigenson, 2011; Starr, Libertus, & Brannon, 2013). Other studies, however, suggest that children's and adults' formal mathematical competencies, whether or not they have learning difficulties, are largely independent of ANS acuity and that individual differences in mathematics achievement are more consistently related to the fluency of processing symbolic numerical and arithmetical information (e.g., Bugden & Ansari, 2011; De Smedt et al., 2011; De Smedt, Noël, Gilmore, & Ansari, 2013; Iuculano, Tang, Hall, & Butterworth, 2008; Rousselle & Noël, 2007) or to more basic processes, such as inhibitory control, that influence performance on both ANS tasks and mathematics achievement tests (Fuhs & McNeil, 2013; Gilmore et al., 2013; but see Keller & Libertus, 2015). To further confuse the matter, Fazio and colleagues (2014) found that ANS acuity and symbolic number knowledge independently contributed to fifth graders' mathematics achievement.

The focus on overall mathematics achievement may have contributed to these mixed results by obscuring potentially more nuanced relations between ANS acuity and mathematical competence (Lourenco, Bonny, Fernandez, & Rao, 2012; Lyons & Beilock, 2011). Early in development, it is possible that ANS acuity contributes to learning some aspects of symbolic mathematics, such as the cardinal value of number words, and once these are understood mathematical learning that builds on this knowledge proceeds independently of the ANS (Chu, vanMarle, & Geary, 2015; Nieder, 2009; vanMarle, Chu, Li, & Geary, 2014). Lourenco and colleagues (2012) found that adults' competence with symbolic arithmetic was related to the acuity of ANS representations of discrete collections of items, whereas competence with symbolic geometry was related to sensitivity to variation in area. The implication is that the relation between ANS acuity and mathematical competence may continue into adulthood but may be specifically related to symbolic competencies in number and arithmetic (see also Park & Brannon, 2013, 2014). It is also the case that people's mathematical education can influence ANS acuity (Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Piazza, Pica, Izard, Spelke, & Dehaene, 2013), but whether this relation is specific to some aspects of mathematics education is