Journal of Experimental Child Psychology 147 (2016) 82–99

Contents lists available at ScienceDirect

Journal of Experimental Child Psychology

journal homepage: www.elsevier.com/locate/jecp

Changing the precision of preschoolers' approximate number system representations changes their symbolic math performance

Jinjing (Jenny) Wang *, Darko Odic, Justin Halberda, Lisa Feigenson

Department of Psychological and Brain Sciences, Johns Hopkins University, Baltimore, MD 21218, USA

A R T I C L E   I N F O

Article history:
Received 30 September 2015
Revised 31 January 2016
Available online 8 April 2016

Keywords:
Numerical cognition
Children
Approximate number system (ANS)
Causal relationship
Symbolic math
Confidence hysteresis

A B S T R A C T

From early in life, humans have access to an approximate number system (ANS) that supports an intuitive sense of numerical quantity. Previous work in both children and adults suggests that individual differences in the precision of ANS representations correlate with symbolic math performance. However, this work has been almost entirely correlational in nature. Here we tested for a causal link between ANS precision and symbolic math performance by asking whether a temporary modulation of ANS precision changes symbolic math performance. First, we replicated a recent finding that 5-year-old children make more precise ANS discriminations when starting with easier trials and gradually progressing to harder ones, compared with the reverse. Next, we show that this brief modulation of ANS precision influenced children's performance on a subsequent symbolic math task but not a vocabulary task. In a supplemental experiment, we present evidence that children who performed ANS discriminations in a random trial order showed intermediate performance on both the ANS task and the symbolic math task, compared with children who made ordered discriminations. Thus, our results point to a specific causal link from the ANS to symbolic math performance.

* Corresponding author.
E-mail address: jenny.wang@jhu.edu (J. Wang).

http://dx.doi.org/10.1016/j.jecp.2016.03.002
0022-0965/

Introduction

Increasing mathematical competence is a central goal of formal education. Yet individuals vary widely in their skill at solving math problems and their comprehension of math concepts. Whereas some children experience great difficulty in mastering mathematical procedures and concepts throughout their lives (Butterworth, Varma, & Laurillard, 2011; Geary, 2004), other children consistently exhibit advanced achievement in mathematics (Brody & Mills, 2005). Given the contribution of mathematics ability to job attainment, salary, and personal debt (Dougherty, 2003; Gerardi, Goette, & Meier, 2013; Parsons & Bynner, 2005; Rivera-Batiz, 1992), there is a pressing need to better understand the sources of individual variability in mathematics comprehension and performance.

One source of variability in mathematical competence is long-term experience with formal mathematics. Differences in the quality and quantity of children's early math learning opportunities have been shown to affect their subsequent math performance (e.g., Beilock, Gunderson, Ramirez, & Levine, 2010; Hill, Rowan, & Ball, 2005; Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010; Maloney, Ramirez, Gunderson, Levine, & Beilock, 2015).

But in addition to the math-related experiences that parents and teachers provide, children also are influenced by an intuitive, non-symbolic, approximate sense of number that is available prior to the onset of schooling (Izard, Sann, Spelke, & Streri, 2009; Lipton & Spelke, 2003; Xu & Spelke, 2000; Xu, Spelke, & Goddard, 2005) and that remains active across the lifespan (Halberda & Feigenson, 2008; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Piazza, Pinel, Le Bihan, & Dehaene, 2007). This number sense is supported by the approximate number system (ANS; for reviews, see Brannon & Merritt, 2011; Dehaene, 1997; Dehaene & Brannon, 2011; Feigenson, Dehaene, & Spelke, 2004; Halberda & Odic, 2014; Nieder & Dehaene, 2009), which is functional in newborn infants (Izard et al., 2009), is used by adults lacking formal math education (Frank, Everett, Fedorenko, & Gibson, 2008; Pica, Lemer, Izard, & Dehaene, 2004), operates across sensory modalities (Barth, Kanwisher, & Spelke, 2003; Feigenson, 2011; Izard et al., 2009; Libertus, Feigenson, & Halberda, 2013b; Nieder, 2012), and has been demonstrated in a wide range of non-human species (Cantlon, Platt, & Brannon, 2009; Dehaene, Dehaene-Lambertz, & Cohen, 1998). The ANS represents numbers in a noisy imprecise fashion, with the imprecision of its numerical representations growing with the target numerosity. Consequently, the ability to nonverbally numerically discriminate two arrays depends on the arrays' ratio rather than their absolute difference (Halberda & Odic, 2014; Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004; Whalen, Gallistel, & Gelman, 1999). For example, discriminating 8 versus 16 dots (a ratio of 2.0) is as easy as discriminating 20 versus 40 dots and is easier than discriminating 32 versus 40 dots (a ratio of 1.25); in this sense, the ANS produces noisy representations that are unlike the exact integers that form the basis of much of the symbolic mathematics that children encounter.

An emerging body of research suggests that, despite the differences between approximate number representations and the exact, symbolically mediated numbers used in school mathematics, the ANS and symbolic math performance are likely related (for reviews, see Chen & Li, 2014; Feigenson, Libertus, & Halberda, 2013; Schneider et al., in press). Evidence in support of this relationship comes from findings that individual differences in ANS precision often correlate with mathematics achievement in children and adults. Performance on standardized math tests has been found to correlate with future ANS ability (Halberda, Mazzocco, & Feigenson, 2008; Libertus, Odic, & Halberda, 2012) and current ANS ability (Bonny & Lourenco, 2013; Inglis, Attridge, Batchelor, & Gilmore, 2011; Libertus, Feigenson, & Halberda, 2011; Linsen, Verschaffel, Reynvoet, & De Smedt, 2014; Lourenco, Bonny, Fernandez, & Rao, 2012; Lyons & Beilock, 2011; Odic et al., 2016), and ANS performance predicts future math ability (Gilmore, McCarthy, & Spelke, 2010; Libertus, Feigenson, & Halberda, 2013a; Mazzocco, Feigenson, & Halberda, 2011b; Starr, Libertus, & Brannon, 2013; van Marle, Chu, Li, & Geary, 2014). In addition, children with mathematical learning disabilities (MLDs or dyscalculia) have significantly poorer ANS precision than typically developing children (Brankaer, Ghesquière, & De Smedt, 2014; Mazzocco, Feigenson, & Halberda, 2011a; Piazza et al., 2010; Skagerlund & Träff, 2016), whereas children with high math achievement show superior ANS precision (Wang & Feigenson, in preparation).