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Brief Report

From rational numbers to algebra: Separable contributions of decimal magnitude and relational understanding of fractions

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ABSTRACT

To understand the development of mathematical cognition and to improve instructional practices, it is critical to identify early predictors of difficulty in learning complex mathematical topics such as algebra. Recent work has shown that performance with fractions on a number line estimation task predicts algebra performance, whereas performance with whole numbers on similar estimation tasks does not. We sought to distinguish more specific precursors to algebra by measuring multiple aspects of knowledge about rational numbers. Because fractions are the first numbers that are relational expressions to which students are exposed, we investigated how understanding the relational bipartite format (a/b) of fractions might connect to later algebra performance. We presented middle school students with a battery of tests designed to measure relational understanding of fractions, procedural knowledge of fractions, and placement of fractions, decimals, and whole numbers onto number lines as well as algebra performance. Multiple regression analyses revealed that the best predictors of algebra performance were measures of relational fraction knowledge and ability to place decimals (not fractions or whole numbers) onto number lines. These findings suggest that at least two specific components of knowledge about rational numbers—relational understanding (best captured by fractions) and grasp of unidimensional magnitude (best captured by decimals)—can be linked to early success with algebraic expressions.

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Introduction

Given the well-documented difficulties that American students often experience in learning algebra and more advanced topics in mathematics (Gonzales et al., 2008; Richland, Stigler, & Holyoak, 2012; Smith & Thompson, 2007), it is important to identify those aspects of earlier mathematics that predict success or failure on advanced topics. Decomposing the prerequisites for success at algebra can potentially guide theoretical analyses of the mental representation of mathematics and also aid in developing more effective instructional strategies. Recent work suggests that knowledge of rational numbers, notably fractions, is closely linked to later success in mathematics. For example, in a large sample of students from the United States and the United Kingdom, Siegler and colleagues (2012) found that fraction knowledge, measured by basic arithmetic and conceptual questions, predicted algebra knowledge and math achievement in general at 16 years of age (beyond what could be predicted from knowledge of whole numbers). Other researchers have also found significant connections between fraction knowledge (including basic conceptual knowledge and performance on tasks that assess grasp of fraction magnitude) and algebra performance (Booth & Newton, 2012; Booth, Newton, & Twiss-Garrity, 2014; Brown & Quinn, 2007; Empson & Levi, 2011).

Although there seems to be an important link between fraction understanding and algebra performance, the nature of this link has yet to be firmly established. Wu (2009) and Siegler, Thompson, and Schneider (2011) emphasized the fact that a fraction, like any other type of number, can be placed on a number line. This understanding requires an integration of procedural and conceptual knowledge about fractions and magnitudes. Two recent studies by Booth and colleagues (Booth & Newton, 2012; Booth et al., 2014) have provided support for this hypothesis. Among middle school students taking elementary algebra classes, a significant correlation was observed between performance on a task requiring estimates of the positions of fractions on a number line and a subsequent algebra test that included problems requiring solving problems, knowledge of critical features in algebraic equations, and coding of equations. Number line estimation with fractions was a stronger predictor of algebra performance than declarative fraction knowledge, a measure of procedural knowledge of how to use fractions in equations, or number line estimation with whole numbers. These findings raise the possibility that a key link between knowledge of rational numbers and algebra performance may involve understanding of fraction magnitudes.

Although understanding of magnitudes is without question a core aspect of mathematical knowledge, there are good reasons to believe that understanding of mathematical *relations* is also critical in grasping algebra. For example, in the algebraic expression $x = 4y$, the value of the variable x is expressed in relation to that of y without any specific magnitude being assigned to either. In recent work (DeWolf, Bassok, & Holyoak, 2015; Rapp, Bassok, DeWolf, & Holyoak, in press), we have emphasized that fractions, with their bipartite a/b structure, naturally convey the relations between the numerator and the denominator (typically two countable sets). Of course, a fraction also represents the magnitude that corresponds to the division of a by b. This duality in the roles of fractions as mathematical representations of relations and magnitudes is similar to the duality of algebraic expressions (Sfard & Linchevski, 1994). Students must understand that they can use algebraic expressions to represent both the relations between quantities and the process used to find an unknown quantity. For example, the quantity of four boxes of equal weight can be represented as $4w$ without knowing the actual magnitude of a box's weight. The expression $4w$ represents the combined weight of the boxes and the process (multiplication) that could be used to determine the total weight given the actual weight of one box. Thus, students' conceptual understanding of fractions as representing both relations and magnitudes may be an important precursor for their subsequent understanding of algebraic expressions.

Interestingly, whereas fractions represent both relations and numerical magnitudes, magnitude equivalent decimals lose the relational structure inherent in a fraction and more directly express one-dimensional magnitude. Studies have shown that magnitude comparisons can be made much more quickly and accurately with decimals than with fractions (DeWolf, Grounds, Bassok, & Holyoak, 2014; Iuculano & Butterworth, 2011) but that fractions are more effective than decimals