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Brief Report

The impact of fraction magnitude knowledge on algebra performance and learning

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A B S T R A C T

Knowledge of fractions is thought to be crucial for success with algebra, but empirical evidence supporting this conjecture is just beginning to emerge. In the current study, Algebra 1 students completed magnitude estimation tasks on three scales (0–1 [fractions], 0–1,000,000, and 0–62,571) just before beginning their unit on equation solving. Results indicated that fraction magnitude knowledge, and not whole number knowledge, was especially related to students' pretest knowledge of equation solving and encoding of equation features. Pretest fraction knowledge was also predictive of students' improvement in equation solving and equation encoding skills. Students' placement of unit fractions (e.g., those with a numerator of 1) was not especially useful for predicting algebra performance and learning in this population. Placement of non-unit fractions was more predictive, suggesting that proportional reasoning skills might be an important link between fraction knowledge and learning algebra.

Introduction

It is widely accepted that fraction knowledge is critical for success in algebra (Empson & Levi, 2011; National Mathematics Advisory Panel [NMAP], 2008), but until recently there has been little empirical evidence to support this claim. Toward this end, Brown and Quinn (2007) found that general fraction knowledge was correlated with algebra knowledge for both elementary and intermediate algebra students. More recently, Siegler and colleagues (2012) found fraction knowledge in upper elementary

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school to be a unique predictor of algebra performance in high school, controlling for variables such as general ability, family income, and whole number knowledge.

Despite this progress, the precise links between fraction and algebra knowledge are unknown. One argument is that fraction skills are used in the process of solving many equations, so automaticity with fraction computation leads to more accurate equation solving (Empson & Levi, 2011). Although this argument is sound, it might not fully explain the relationships described above. In addition to proficiency with computation, some scholars argue that a deep understanding of numbers and number properties may be crucial for success in algebra (Empson & Levi, 2011; Ketterlin-Geller & Chard, 2011; Wu, 2001).

One number property that is central to a deep understanding of numbers, including both whole numbers and fractions, is magnitude (Siegler, Thompson, & Schneider, 2011). The understanding of whole number magnitudes, or relative sizes of numbers, has been linked to arithmetic knowledge, learning of arithmetic, and overall mathematics performance (Booth & Siegler, 2006, 2008). Similarly, fraction magnitude knowledge predicts both fraction arithmetic knowledge and overall mathematics performance (Siegler et al., 2011). It has also been empirically linked to algebra readiness. In particular, Booth and Newton (2012) found evidence that knowledge of fraction magnitudes, as evidenced by the ability to estimate placement of unit fractions on a number line, was related to equation solving for students prepared to take Algebra 1. In addition, it was related to conceptual knowledge and the ability to solve algebra word problems, supporting the notion that there is more than automaticity linking knowledge of fractions and algebra. However, participants in this study were not yet learning algebra; thus, replication of these findings with students enrolled in an algebra course is necessary.

That fraction magnitude is determined by a relationship *between* two numbers is an important aspect of fraction knowledge that takes time to fully understand (Siegler & Pyke, in press; Stafylidou & Vosniadou, 2004). Research suggests that even adults fail to process this relationship automatically and, therefore, struggle with accuracy and speed on tasks that require its application (Bonato, Fabbri, Umiltà, & Zorzi, 2007; Fabbri, Caviola, Tang, Zorzi, & Butterworth, 2012; Iuculano & Butterworth, 2011). Yet this understanding is critical for developing the concepts of fraction equivalence and proportionality, which may play roles in algebra learning. For example, "All fractional representations of 1/3 will fit into the equation $x/y = 1/3$, which is equivalent to the equation $y = 3x$" (Empson & Levi, 2011, p. 134). Using this view of linear equations, Cetin and Ertekin (2011) examined the relationship between proportional reasoning and equation solving in a study of eighth graders in Turkey, finding that proportional reasoning was significantly correlated with success in equation solving. Booth and Newton (2012), however, found only a marginal correlation between equation solving and number line estimates reflecting students' proportional reasoning skills. As students begin the study of algebra, this type of reasoning may become more important.

The current study

The current study had three main purposes. First, it aimed to replicate and extend Booth and Newton's (2012) findings on the relation of fraction and whole number magnitude knowledge on algebra skills for students taking Algebra 1. Students were tested on conceptual and procedural knowledge of solving two-step linear equations as well as on their ability to encode equations. Because students were older, larger whole number scales were used. This is critical because even younger students have been shown to be proficient on the scales typically used to measure number line estimation (e.g., students produce linear estimates on 0–1000 scales by fourth grade [Booth & Siegler, 2006]); thus, there would be insufficient variance to distinguish between those with greater and lesser whole number magnitude knowledge. Larger scales have previously been used successfully with older students and adults (e.g., Siegler, Thompson, & Opfer, 2009).

Second, how do fraction and whole number magnitude knowledge predict *learning* in algebra? Because we do not believe that the role of fraction knowledge in algebra performance is limited to automaticity benefits, we expect to find relations between fraction magnitude knowledge and learning in equation solving as well as in measures of conceptual understanding. Given that prior work used general fraction knowledge (Brown & Quinn, 2007; Siegler et al., 2012), we aimed to begin