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A combined economic analysis of optimal planting density, thinning and rotation for an even-aged forest stand

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ABSTRACT

A planting and timber harvest scheduling model for even-aged forest stands was developed by combining and extending the dynamic thinning approach and existing work on planting density. A net present value maximum solution for planted volume, thinning regime and rotation schedule was determined simultaneously. The influence of the planting density on the optimal stand volume path, thinning schedule and rotation length was analyzed, emphasizing the importance of optimal stand establishment. The results and dependencies are discussed in detail and compared to findings presented in the literature.

1. Introduction

Since the beginnings of silvicultural research, forest economists have studied the relationships between planting density, thinning and rotation length, and their influence on the profitability of forest stands (cf. Thünen, 1875/ 2009).

However, not until the Faustmann revival (Amacher et al., 2009: 7) did a rich stream of analyses shed any light on the complex problem of silvicultural treatments. One group of studies analyzed the dependencies between thinning and rotation length. Most applied dynamic programming or Pontryagin's maximum principle; for example, Näslund (1969), Kilkki and Vaisanen (1969), Schreuder (1971), Clark and De Pree (1979), Kao and Brodie (1980), Cawrse et al. (1984), Betters et al. (1991), Borchert (2002), Hyytiainen and Tahvonen (2002), and many others. Two publications by Coordes (2013a, 2014) analyzed these dependencies by applying a kind of stand unit model based on single-tree growth.

In another group of papers, the dependencies between planting density and rotation length were analyzed; for example, Chang (1983), Zhou (1999), Coordes (2013b), and others. Hyytiainen et al. (2005) and Cao et al. (2006) studied the relationships between these three components with the help of numerical optimization. Gong (1998) provided a numerical evaluation of a general decision model to determine the optimal planting density and its effect on harvest and rotation under different price scenarios.

However, there exists no qualitative economic analysis of even-aged forest stand management in which planting, thinning and rotation are studied together in the same model. The aim of this study, therefore, was to carry out an economic analysis of the relationships between planting density, thinning and rotation length in order to understand their simultaneous effect on the land expectation value (LEV), as a contribution to close the gap between the two separate views on planting and thinning. A further intention was to enrich the existing knowledge concerning the optimal thinning solution by providing a more detailed insight into the characteristics and dependencies of the optimal timber stock path.

As a first step, an extended Faustmann model was developed to incorporate the interactions of the three silvicultural components mentioned above. The approach for the modeling of the simultaneous effect of thinning and rotation length followed Clark and De Pree (1979) and Cawrse et al. (1984).

Expanding upon to their approach, (I) a planting density was inserted, (II) a density dependent timber price process was introduced, and (III) a more general timber stock density model was employed. In contrast to their approach, different unit net revenue levels for thinning and clearcutting were not used. A satisfactory analysis of this effect is provided in the two papers mentioned above and further research is not required. The consequence for the model presented here is greater transparency.

The research presented did not end with solving the threefold simultaneous optimization problem. In a second step, the characteristics

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of the optimal timber stock path and rotation length are analyzed with special regard to planting density and, subsequently, the findings are discussed in the context of the results of related papers. Finally, some general insights into the regeneration and exploitation of even-aged forest stands as a natural timber resource are provided.

2. Model

Consider a deterministic forest growth model, where the timber stock $Q(t)$ of a t-year-old stand is evolving with an age- and volume-dependent annual increment $\phi(t, Q(t))$ and is reduced by a nonnegative harvest $h(t)$. The stand volume development process follows a differential equation given by

$$\frac{dQ}{dt} = \phi(t, Q) - h(t).$$

Let $Q(0) = Q_0 > 0$ represent the initial volume at planting (stocking density) with planting costs $C(Q_0)$. Assuming that the growth potential of a stand decreases with age, we set $\partial\phi/\partial t \leq 0$. The stand's increment ϕ is assumed to be concave with respect to stand volume Q. The assumption for this is that the absolute volume increment increases with stand volume until some ϕ-maximum timber stock level $\hat{Q}(t)$. Above this level, the influence of growth competition starts to reduce the increment. In this sense, $\hat{Q}$ may be regarded a competition border, with $\partial\hat{Q}/\partial t > 0$. We assume $\partial\phi/\partial Q > 0$ for $Q < \hat{Q}(t)$, $\partial\phi/\partial Q = 0$ for $Q = (t)$ and $\partial\phi/\partial Q < 0$ for $Q > \hat{Q}(t)$. Furthermore, we assume $\partial^2\phi/\partial Q^2 < 0$. We refer to this term as the 'density effect' on competition because it describes the influence of a marginal change in stand density on the volume impact on the stand's increment.

These assumptions are illustrated in Fig. 1.

The volume influence $\partial\phi/\partial Q$ on the yearly increment depends on the stand age. Younger stands usually react more strongly to competition and, therefore, to density reduction. This ability declines with increasing stand age. We refer to $\partial^2\phi/\partial Q\partial t$ as the 'age effect' on competition. We assume the impact of density, the density effect, to be dominant with

$$\left|\frac{\partial^2\phi}{\partial Q^2}\right| > \left|\frac{\partial^2\phi}{\partial Q\partial t}\right|.$$

All assumptions are summarized in Fig. 2 for a comparison of two stand ages $t_1 < t_2$. The younger stand has a steeper curve due to the age effect.

Let $p(t, Q_0)$ be the net revenue for one volume unit of timber. It is assumed to be a function of stand age and planted volume Q_0. Stand age is used as a proxy for increasing single tree volumes and, therefore,

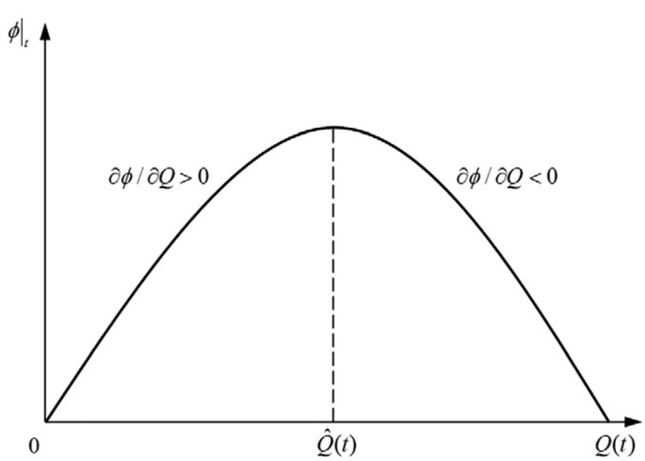

Fig. 1. Density and competition effect.

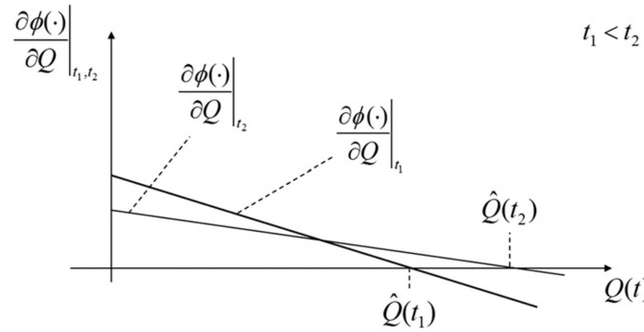

Fig. 2. Increment impact of stand age and stand volume.

decreasing harvest costs. Thus, we assume $\dot{p} > 0$. In addition, the single tree volume of a homogenous stand depends on stand density. High stand density usually results in low tree volumes and, correspondingly, a low net price level for timber. Two effects might impact upon the stand density. First, thinnings reduce the stand volume and may, therefore, have a positive impact on single tree volumes. Second, the planting density has a great influence on the stand density. Planting is a main factor determining tree volume, especially in situations where thinnings are not optimal, or only occur at higher ages. In this model we focus on the effects of planting and, therefore, ignore the possible influence of thinnings on the price level. We assume $\partial p(t, Q_0)/\partial Q_0 < 0$.

Although net timber prices are usually convex with respect to stand age, we will use a linear price function of the form $p(t, Q_0)) = at + b(Q_0)$, with $a > 0$, $b < 0$ and $\partial b(Q_0)/\partial Q_0 < 0$. This is a simplification greatly reducing the complexity of the model without affecting the general results.

The only goal of forest management is the maximization of the net present value of all future timber production cash flows, which we call LEV, given a constant interest rate r.

In this model, the state of the dynamic system at age t is described by the timber stock $Q(t)$. The thinning volume h(t) is used to control the stand volume.

The optimization problem with regard to finding the optimal initial volume at planting Q_0^*, thinning strategy $h^*(t)$ and rotation length T^* is given by:

$$\max_{h(t),T,Q_0} \int_0^T \left(1-e^{-rT}\right)^{-1} e^{-rt} p(t, Q_0) h(t) dt + \left(1-e^{-rT}\right)^{-1} \left[-C(Q_0) + e^{-rT} p(T, Q_0) Q(T)\right]$$

subject to:

$$\begin{aligned}&\frac{dQ}{dt} = \phi(t, Q) - h(t)\\&0 \leq h(t) \leq Q(t)\\&Q_0 > 0.\end{aligned} \qquad (01)$$

The optimal set of variables $\{Q_0^*, h^*(t), T^*\}$ maximizes the LEV. The optimal solution is restricted by the growth function and a budget constraint for the thinning volume $h(t)$.

The Lagrangian of the dynamic problem is:

$$L = \int_0^T \left[\left(1-e^{-rT}\right)^{-1} e^{-rt} p(t, Q_0) h(t) + \lambda(t)\left[\phi(t, Q) - h(t) - \dot{Q}\right]\right] dt + \left(1-e^{-rT}\right)^{-1} \left[-C(Q_0) + e^{-rT} p(T, Q_0) Q(T)\right].$$