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Patterns of strengths and weaknesses in children's knowledge about fractions

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ABSTRACT

The purpose of this study was to explore individual patterns of strengths and weaknesses in children's mathematical knowledge about common fractions. Tasks that primarily measure either conceptual or procedural aspects of mathematical knowledge were assessed with the same children in their fourth- and fifth-grade years ($N = 181$, 56% female and 44% male). Procedural knowledge was regressed on levels of conceptual knowledge, and vice versa, to obtain residual scores. Residual scores capture variability in each kind of math knowledge that is not shared with the other type of knowledge. Cluster analysis using residuals indicated four distinct knowledge profiles in fourth graders: (a) higher than expected conceptual knowledge and relatively lower procedural knowledge, (b) relatively lower conceptual knowledge and higher procedural knowledge, (c) lower concepts but expected levels of procedural knowledge, and (d) relatively higher than expected levels of both procedural and conceptual knowledge. In fifth grade, another cluster emerged that showed lower procedures but expected levels of conceptual knowledge. In general, students with relatively lower than expected conceptual knowledge showed poorer accuracy on measures used to form the clusters and also word problem setups and estimation of sums. Implications for explaining seemingly conflicting results from prior work across studies are discussed.

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Introduction

Mathematics involves a variety of related types of knowledge (e.g., Bransford, Brown, & Cocking, 2000; Greeno, 1991; Hecht, 1998). Two distinct types of knowledge that are essential for mathematical competence are conceptual understanding and procedural skill (Hiebert & Lefevre, 1986; Moss & Case, 1999; National Mathematics Advisory Panel [NMAP], 2008; Schneider & Stern, 2010). Conceptual knowledge involves meaningful understanding about underlying principles that govern a domain, and procedural knowledge is awareness of the processing steps or algorithms for solving a problem (Bisanz & LeFevre, 1990; Rittle-Johnson, Siegler, & Alibali, 2001). Conceptual understanding could be used during the process of solving 1/2 + 1/4 by shading corresponding regions of a circle or using a number line (Cramer, Wyberg, & Leavitt, 2008). Procedural knowledge could be used to convert the addends into common denominators and then add the numerators. Procedural steps can be carried out either with or without understanding why the algorithm works. Indeed, some children might have poor representations of the magnitudes of fraction symbols and, therefore, resort to using procedures in a mechanical way (Geary, 1994; Rittle-Johnson & Siegler, 1998). Little is understood about how these types of knowledge are related to each other, such as how children's performance on one type of mathematical knowledge corresponds to their accuracy on the other type of knowledge. It is also unclear whether particular strengths and weaknesses in mathematical knowledge are related to achievement. Such understanding would provide insight concerning the nature of mathematical knowledge and practical guidance about how to identify and treat children with math difficulties (MD). The purpose of the current study was to explore individual patterns of strengths and weaknesses in children's procedural and conceptual knowledge. We examined this issue with respect to emerging fraction skills, a domain of math that children and adults find to be particularly difficult (Hecht & Vagi, 2010; NMAP, 2008).

The ability to accurately represent number magnitudes is a central characteristic of conceptual knowledge about numbers (Hecht, Vagi, & Torgesen, 2007; Hiebert & Lefevre, 1986; Kilpatrick, Swafford, & Findell, 2001; Siegler, Thompson, & Schneider, in press). For example, 1/2 can refer to a pie with half of it eaten, two pies with one of them eaten, and so on. Part–whole knowledge can also be used to determine the relative size of fraction numerals (Cramer, Post, & del Mas, 2002), such as ordering fractions from lowest to highest (e.g., Hecht, Close, & Santisi, 2003; Mazzocco & Devlin, 2008; Smith, Solomon, & Carey, 2005). Computation can be used to measure procedural knowledge because a step-by-step procedure is presumably the dominant approach for finding the exact answer (cf. Byrnes & Wasik, 1991; Hallett, Nunes, & Bryant, 2010; Hecht, 1998; Hiebert & Lefevre, 1986; Kerslake, 1986; Rittle-Johnson & Siegler, 1998).

Despite considerable research focusing on conceptual and procedural knowledge in mathematical cognition (Canobi, 2004; Gilmore & Papadatou-Pastou, 2009; Rittle-Johnson & Siegler, 1998), there has not yet emerged a universal and agreed-on way to measure each type of knowledge independently of the other type. That is, performance on a task that presumably measures one type of knowledge may involve the other kind of math knowledge to some extent (Schneider & Stern, 2010). Accordingly, associations between concepts and procedures are reported within the range of .4 to .6 (e.g., Byrnes & Wasik, 1991; Hallett et al., 2010; Hecht, 1998), and performance on each type of knowledge predicts growth in the other type (e.g., Hecht & Vagi, 2010; Rittle-Johnson & Alibali, 1999). Conceptual knowledge might be used when performing step-by-step procedures, such as by using knowledge about fraction symbol magnitudes to check whether or not the answer to a problem is plausible (Hiebert & Lefevre, 1986). Likewise, conceptual knowledge tasks might involve rotely learned procedural steps, such as memorizing the rule to determine the number of regions to shade based on the size of the denominator or combining the amounts depicted by two area models by converting each picture into a fraction symbol and then using a procedure to add up the fractions. Consequently, methods are needed to estimate students' ability on one type of knowledge that is independent of the other kind of knowledge.

Hallett and colleagues (2010) demonstrated a means to obtain purer estimates of procedural and conceptual knowledge based on residual scores obtained from regression analysis. The goal was to obtain a conceptual knowledge variable that was maximally different from the procedural knowledge task and vice versa. Conceptual items were judged by two of the authors of Hallett and colleagues to