Journal of Experimental Child Psychology 103 (2009) 532–545

Contents lists available at ScienceDirect

Journal of Experimental Child Psychology

journal homepage: www.elsevier.com/locate/jecp

Children's understanding of addition and subtraction concepts

Katherine M. Robinson *, Adam K. Dubé

Department of Psychology, Campion College, University of Regina, 3737 Wascana Parkway, Regina, Sask., Canada S4S 0A2

ARTICLE INFO

Article history:
Received 28 July 2008
Revised 3 December 2008
Available online 22 January 2009

Keywords:
Arithmetic
Inversion
Associativity
Conceptual knowledge
Addition
Subtraction

ABSTRACT

After the onset of formal schooling, little is known about the development of children's understanding of the arithmetic concepts of inversion and associativity. On problems of the form $a + b - b$ (e.g., $3 + 26 - 26$), if children understand the inversion concept (i.e., that addition and subtraction are inverse operations), then no calculations are needed to solve the problem. On problems of the form $a + b - c$ (e.g., $3 + 27 - 23$), if children understand the associativity concept (i.e., that the addition and subtraction can be solved in any order), then the second part of the problem can be solved first. Children in Grades 2, 3, and 4 solved both types of problems and then were given a demonstration of how to apply both concepts. Approval of each concept and preference of a conceptual approach versus an algorithmic approach were measured. Few grade differences were found on either task. Conceptual understanding was greater for inversion than for associativity on both tasks. Clusters of participants in all grades showed that some had strong understanding of both concepts, some had strong understanding of the inversion concept only, and others had weak understanding of both concepts. The findings highlight the lack of developmental increases and the large individual differences in conceptual understanding on two arithmetic concepts during the early school years.

Introduction

The study of children's mathematical cognition is increasingly focused on the development of children's arithmetic concepts. Arithmetic concepts are integral in developing proficiency in math-

* Corresponding author. Fax: +1 306 359 1200.
E-mail address: katherine.robinson@uregina.ca (K.M. Robinson).

0022-0965/$ - see front matter Crown Copyright © 2008 Published by Elsevier Inc. All rights reserved.
doi:10.1016/j.jecp.2008.12.002

ematics (National Council of Teachers of Mathematics [NCTM], 2000). Understanding the relation between the operations of addition and subtraction is necessary for children's knowledge of the additive components of number and part–whole relations (Baroody & Lai, 2007), but this understanding is a difficult conceptual achievement (Canobi, 2005). Two specific concepts of arithmetic that depend on the understanding of the relationship between addition and subtraction are inversion and associativity. Both concepts can be assessed through problem solving (e.g., Robinson, Ninowski, & Gray, 2006). On problems of the form $a + b - b$, if children understand that addition and subtraction are inverse operations and apply that information, then problem solving is fast and accurate because no calculations are required (Bisanz & LeFevre, 1990). On problems of the form $a + b - c$, if children understand that the answer will be the same no matter which pair of numbers is dealt with first (e.g., $[3 + 97] - 92 = 3 + [97 - 92]$) (Canobi & Bethune, 2008; Canobi, Reeve, & Pattison, 1998), then problem solving can be faster and more accurate than using a left-to-right algorithmic approach on many problems even if computation is still required (Robinson et al., 2006). The inversion concept in particular has been examined by several researchers, often in regard to how early the concept is acquired by children (e.g., Sherman & Bisanz, 2007). In contrast, little research has been conducted on the concept of associativity, particularly on problems involving two arithmetic operations.

The application of both the inversion concept and the associativity concept in problem solving is dependent on consideration of the entire problem rather than application of a left-to-right problem-solving approach. Siegler and Araya's (2005) SCADS$*$ model proposes that the inversion shortcut—simply stating that the answer is the first number because the second and third numbers cancel each other out rather than performing any calculations—is dependent on a participant shifting attention to and encoding the $b - b$ portion of the problem. Subsequently, the inversion shortcut would be primed and, if the strength of association were strong enough, would be activated and used. Similarly, the associativity shortcut—solving $b - c$ and then adding the result to a—would also be dependent on shifting attention to and encoding the latter part of the problem for subsequent shortcut priming and activation to occur. Thus, although the inversion concept focuses on children's understanding of the inverse relationship between addition and subtraction and the associativity concept focuses on the understanding that addition and subtraction can be done in any order, if the concepts are to be applied to problem solving, then participants must pay particular attention to the right side of the problem rather than dealing with the problem using the typical left-to-right approach.

The inversion concept

Most recent studies of the inversion concept have focused on how early children are able to understand the inversion concept. Studies on preschoolers have shown that formal instruction is not necessary to understand the inversion concept, although not all children demonstrate understanding (e.g., Baroody & Lai, 2007; Gilmore & Spelke, 2008; Klein & Bisanz, 2000; Sherman & Bisanz, 2007). A number of studies have also examined school-aged children's understanding of the concept (e.g., Bryant, Christie, & Rendu, 1999; Gilmore, 2006; Gilmore & Bryant, 2006; Robinson et al., 2006; Siegler & Stern, 1998; Stern, 1992). Most studies have not focused on age-related differences after the preschool years. By adulthood, most individuals use the inversion concept (Robinson & Ninowski, 2003), but the developmental pathway after the beginning of formal schooling has rarely been examined. Canobi and Bethune (2008) found that 6-year-olds were faster and more accurate on inversion problems than 5-year-olds, suggesting that school-aged children were using the inversion shortcut more frequently than preschoolers. In an unpublished study, Bisanz, LeFevre, and Gilliland (1989), using verbal reports, found that 29% of both 7- and 9-year-olds used the inversion concept, whereas 58% of 11-year-olds used the concept. Stern (1992) used differences in solution latencies to determine that 13%, 35%, and 45% of 8-, 9-, and 10-year-olds, respectively, used the inversion shortcut when presented with inversion and standard problems mixed together. However, the development of the concept was not the main focus of that study, so whether there were significant age-related differences was not examined. Overall, relatively little is known about how the inversion concept develops after the preschool period.