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The association between symbolic and nonsymbolic numerical magnitude processing and mental versus algorithmic subtraction in adults☆

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ABSTRACT

There are two well-known computation methods for solving multi-digit subtraction items, namely mental and algorithmic computation. It has been contended that mental and algorithmic computation differentially rely on numerical magnitude processing, an assumption that has already been examined in children, but not yet in adults. Therefore, in this study, we examined how numerical magnitude processing was associated with mental and algorithmic computation, and whether this association with numerical magnitude processing was different for mental versus algorithmic computation. We also investigated whether the association between numerical magnitude processing and mental and algorithmic computation differed for measures of symbolic versus nonsymbolic numerical magnitude processing. Results showed that symbolic, and not nonsymbolic, numerical magnitude processing was associated with mental computation, but not with algorithmic computation. Additional analyses showed, however, that the size of this association with symbolic numerical magnitude processing was not significantly different for mental and algorithmic computation. We also tried to further clarify the association between numerical magnitude processing and complex calculation by also including relevant arithmetical subskills, i.e. arithmetic facts, needed for complex calculation that are also known to be dependent on numerical magnitude processing. Results showed that the associations between symbolic numerical magnitude processing and mental and algorithmic computation were fully explained by individual differences in elementary arithmetic fact knowledge.

1. Introduction

There are generally two well-known computation methods for solving multi-digit arithmetic items, namely mental and algorithmic computation. Mental computation can be defined as performing arithmetic operations on numbers (e.g., solving $57 - 34$ as $57 - 30 = 27$, $27 - 4 = 23$) and is typically done in one's head, while in algorithmic computation one operates on digits (e.g., solving $57 - 34$ as $5 - 3 = 2$, $7 - 4 = 3$), and this is typically done with paper and pencil. A common assumption among mathematics educators is that these two computation methods differ in the extent to which they rely on basic number sense (Thompson, 1999; Van den Heuvel-Panhuizen, 2001; Verschaffel, Greer, and De Corte, 2007). More precisely, it has been contended that mental computation is more strongly associated with basic number sense than algorithmic computation. A recent study in fourth graders (Linsen, Verschaffel, Reynvoet, and De Smedt, 2015) showed that children's numerical magnitude processing, which is a prominent aspect of number sense, was more strongly associated with mental computation than with algorithmic computation, although the latter was also significantly associated with children's numerical magnitude processing. Although many studies (e.g. De Smedt, Noël, Gilmore, and Ansari, 2013, for a review) have focused on the association between numerical magnitude processing and mathematical achievement (for general as well as for more specific aspects of mathematical achievement) in children, little is known about this association in adults. Because children and adults largely differ in their experience with both mental and algorithmic computation, results from studies with children cannot be merely generalized to adults. Therefore, we investigated whether the findings from the study of Linsen et al. (2015) could be replicated in adults. Other than that, this knowledge adds to the literature in two distinct ways. Firstly, many researchers who focused on the role of numerical magnitude processing in mathematical achievement have only included general mathematical achievement tests. Scores on these tests, however, reflect performance on a broad and unspecified range of mathematical skills, while there is no theoretical reason to assume that numerical magnitude processing is equally important, or even important at all, for all different aspects of mathematical achievement.

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Therefore, looking at different specific aspects of mathematical achievement can provide us with more detailed information on this association. Secondly, it is important to also include other meaningful factors when exploring the association between numerical magnitude processing and aspects of mathematical achievement. Elementary arithmetic fact knowledge has been shown to be associated with numerical magnitude processing (Vanbinst, Ghesquière, and De Smedt, 2012, in children) and is, moreover, crucial for multi-digit computation (Cornoldi and Lucangeli, 2004). For this reason, we included an elementary arithmetic fact task and examined its possible mediating role in the association between numerical magnitude processing and multi-digit computation.

1.1. Numerical magnitude processing

Basic number sense or the ability to process numerical magnitudes (Berch, 2005) is most commonly assessed with a numerical magnitude comparison task (see De Smedt et al., 2013, for a review). In this task, one is instructed to indicate the numerically larger of two presented numerical magnitudes. The numerical magnitude comparison task can be assessed in a symbolic format, using Arabic digits, or a nonsymbolic format, using dots (e.g., Castronovo and Göbel, 2012; Lyons and Beilock, 2011) and both single-digit and double-digit stimuli can be used (e.g., Linsen, Verschaffel, Reynvoet, and De Smedt, 2014). Previous research with adults has shown that performance on this task is associated with individual differences in mathematical skills, assessed with general standardized math achievement tests (see De Smedt et al., 2013, for a review; see Chen and Li, 2014, and Fazio, Bailey, Thompson, and Siegler, 2014, for two meta-analyses), but results differ depending on the number format that is used.

For the nonsymbolic format, a range of studies found a significant association between nonsymbolic numerical magnitude processing and mathematical achievement (Halberda, Ly, Wilmer, Naiman, and Germine, 2012; Libertus, Odic, and Halberda, 2012; Lourenco, Bonny, Fernandez, and Rao, 2012; Lyons and Beilock, 2011), while others did not (Castronovo and Göbel, 2012; Inglis, Attridge, Batchelor, and Gilmore, 2011; Price, Palmer, Battista, and Ansari, 2012). The above-mentioned meta-analysis by Chen and Li (2014) showed that there is a small but statistically significant association ($r = 0.20$, 95% CI = [0.14, 0.26]), between the accuracy in nonsymbolic numerical magnitude processing and math performance. Chen and Li (2014) further suggested that this association is unaffected by age and still exists in adults. The other meta-analysis by Fazio et al. (2014) also led to the conclusion that the association between nonsymbolic numerical magnitude processing and math achievement is small ($r = 0.22$, 95% CI = [0.20, 0.25]), but significant. They also evaluated the influence of several potential moderators, including age. Results showed that the association between nonsymbolic numerical magnitude processing and mathematical achievement tends to be weaker for children beyond 6 years of age and for adults. This difference between the two meta-analyses concerning the effect of age on the association between nonsymbolic numerical magnitude processing and mathematical achievement might be due to the different inclusion criteria for the studies that were involved and the different ways of analyzing the effects of age.

For the symbolic format, only two studies have examined its association with mathematical achievement in adults (Castronovo and Göbel, 2012; Lyons and Beilock, 2011) and yet no meta-analysis that statistically summarizes this association for children nor adults has been published, although a narrative review by De Smedt et al. (2013) suggested that this association seemed to be more robust than with the nonsymbolic format. Results of Castronovo and Göbel (2012) and Lyons and Beilock (2011) consistently showed that adults who were faster in indicating the larger of two Arabic digits had higher mathematics achievement.

The distinction between the symbolic and nonsymbolic formats of the numerical magnitude comparison task relates to the debate whether the representation of numerical magnitudes per se, or its access via symbolic digits, is important for mathematical achievement. When numerical magnitude processing per se is crucial for mathematics achievement, one expects both symbolic and nonsymbolic tasks to predict individual differences in mathematics achievement. If, on the other hand, only symbolic tasks predict individual differences in mathematical achievement, the idea that the access to numerical meaning from symbolic digits is important is favored. Although this issue has been examined extensively in children (De Smedt and Gilmore, 2011; Holloway and Ansari, 2009; Landerl and Kölle, 2009; Lonnemann, Linkersdörfer, Hasselhorn, and Lindberg, 2011; Mussolin, Mejias, and Noël, 2010; Rousselle and Noël, 2007; Sasanguie, De Smedt, Defever, and Reynvoet, 2012; Vanbinst et al., 2012; see De Smedt et al., 2013, for a review), only two studies in adults administered both a symbolic and a nonsymbolic numerical magnitude comparison task (Castronovo and Göbel, 2012; Lyons and Beilock, 2011). Results of these two studies were inconsistent. Castronovo and Göbel (2012) investigated 71 university students' mathematics achievement with the arithmetic subtest of the fourth Wide Range Achievement Test (WRAT4) and speeded calculation exercises (i.e., additions, subtractions and multiplications). Results showed that only symbolic and not nonsymbolic numerical magnitude processing was associated with mathematical achievement. On the other hand, Lyons and Beilock (2011) observed that both symbolic and nonsymbolic numerical magnitude processing were associated with mathematical achievement as measured with a mental arithmetic task (addition, subtraction, multiplication and division) in a study with 54 university students. The inconsistency in results might be explained by the task that was used to measure participants' mathematical achievement, as depending on the mathematical content of the task the importance of numerical magnitude processing might be smaller or larger.

This idea that the association between numerical magnitude processing and mathematical achievement is more important for some aspects of mathematical performance than others has already been explored in several studies with children (Linsen et al., 2014; Linsen et al., 2015; Vanbinst et al., 2012). However, adult studies on this topic are sparse and moreover, the existing studies, which focused on geometry (Lourenco et al., 2012) and various subtests of the Woodcock–Johnson III Tests of Achievement (Inglis et al., 2011; Lourenco et al., 2012), only included nonsymbolic measures of numerical magnitude processing. It thus remained to be determined how symbolic numerical magnitude processing is associated with specific aspects of mathematical performance.

Extending the existing body of evidence, this study therefore examined the association between nonsymbolic and symbolic numerical magnitude processing and two specific aspects of mathematical performance, namely mental and algorithmic computation. The focus on these aspects was driven by the assumption made by several math educators (Thompson, 1999; Van den Heuvel-Panhuizen, 2001; Verschaffel et al., 2007), who assume that mental and algorithmic computation differ in their reliance on numerical magnitude processing, based on the specific characteristics of these computation methods. In the following we will explain these computation methods in detail.

1.2. Mental versus algorithmic computation

There are two well-known computation methods for solving multi-digit subtraction, namely mental and algorithmic computation, which differ on the following four essential characteristics. Firstly, algorithmic computation consists of a fixed sequence of well-defined and elementary calculation steps, executed on the *digits* (rather than the numbers) of the problem ($78 - 26 = ?$; $7 - 2 = 5$ and $8 - 6 = 2$, so the answer is 52). In mental computation, on the other hand, one does not calculate with digits but with the *numbers* in the problem ($78 - 26 = ?$; $78 - 20 = 58$ and $58 - 6 = 52$). Secondly, in algorithmic computation there is a strict solution path that can be followed to guarantee a correct