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When you don't have to be exact: Investigating computational estimation skills with a comparison task

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ABSTRACT

The present study is the first systematic investigation of computational estimation skills of multi-digit multiplication problems using an estimation comparison task. In two experiments, participants judged whether an estimated answer to a multi-digit multiplication problem was larger or smaller than a given reference number. Performance was superior in terms of speed and accuracy for smaller problem sizes, for trials in which the reference numbers were smaller vs. larger than the exact answers (consistent with the size effect) and for trials in which the reference numbers were numerically far compared to close to the exact answers (consistent with the distance effect). Strategy analysis showed that two main strategies were used to solve this task—approximate calculation and sense of magnitude. Most participants reported using the two strategies. Strategy choice was influenced by the distance between the reference number and exact answer, and by the interaction of problem size and reference number size. Theoretical implications as to the nature of numerical representations in the ANS (approximate number system) and to the estimation processes are suggested.

1. Introduction

Imagine a situation where you are searching the internet for a hotel in Paris. You wish to book a hotel room for your 24-night stay, and can spend about 1000$ on accommodation. You find an advertisement for an appealing, well-located hotel charging 85$ per night. Can you afford it? To answer this question, the information should be translated into an arithmetic problem (85×24). To solve it exactly you would most likely need a calculator or at least a paper and pencil. However, in such a situation you do not really need to solve this problem exactly—an approximate response is sufficient for your purposes. In fact, all you need to know is whether the answer to this multiplication problem is smaller or larger than the 1000$ you can spend on accommodation. Although such a situation is common in everyday life, surprisingly little research has been devoted to the investigation of how people approach such problems.

Putting it in a more general context, this is a case of computational estimation, which is the process of producing an approximate answer to an arithmetic problem. Despite its usefulness in a variety of circumstances, a relatively small number of studies have looked into computational estimation skills (e.g., Dowker, 1997, 2003; Dowker, Flood, Griffiths, Harriss, & Hook, 1996; Lemaire & Lecacheur, 2002; Lemaire, Lecacheur, & Farioli, 2000; Levine, 1982; Rubinstein, 1985; Sowder & Wheeler, 1989). Past studies mainly used the *estimation production task* (e.g., Imbo & LeFevre, 2011; LeFevre, Greenham, & Waheed, 1993; Lemaire, Arnaud, & Lecacheur, 2004; Lemaire & Lecacheur, 2002, 2010; Lemaire et al., 2000). In this task participants are presented with an arithmetic problem and they are asked to generate an estimate for the answer. Such studies concluded that although estimation skills improve with age (e.g., Dowker, 1997) and numerical skills (e.g., Dowker et al., 1996), children are surprisingly bad estimators, and even many adults are far from good at it (see Siegler & Booth, 2005 for a review), as indicated for example, by their frequent place value errors (i.e., estimates that are ten times smaller or larger than the exact answer) (Ganor-Stern & Siegler, 2004; LeFevre et al., 1993).

Yet in many cases, such as the one described previously, it is not really necessary to produce an estimate for an arithmetic problem, but rather it is sufficient to judge whether the result of such a problem is larger or smaller than a certain number. Little is known about how people approach such a task.

A series of works examined the estimation of arithmetic operations with non-symbolic stimuli. McCrink, Dehaene, and Dehaene-Lambertz (2007) presented sets of objects being added or subtracted from one another and participants had to judge whether the final numerosity was correct or incorrect. When using quantities of up to 30, the mean estimates of participants were at the true numerical outcome.

Past studies have shown that even 5-year-old children can estimate the results of simple arithmetic operations with non-symbolic

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numerosities (Barth, La Mont, Lipton, & Spelke, 2005; Barth et al., 2006; Gilmore, McCarthy, & Spelke, 2010). When asked to compare the sum of two quantities in the range of 5 to 58 with a reference quantity, their performance was above chance. McCrink and Spelke (2010) extended this line of research and examined the ability of 5–7-year-old children to estimate the result of multiplication of numerosities.

Evidence for children approximate arithmetic of quantities presented in a symbolic format was provided by Gilmore, McCarthy, and Spelke (2007). Five-to-six year-old children before formal instruction of arithmetic performed above chance level when asked to compare the estimated results of addition and subtraction problems of numbers in the range of 5–98 to a reference number.

Importantly, in all the above mentioned studies performance was affected by the ratio between the result of the arithmetic problem and the compared quantity, with better performance for larger ratios. This ratio effect is considered a signature of the ANS (approximate number system) the system for representing large, approximate quantities (Feigenson, Dehaene, & Spelke, 2004). This system is shared by humans and nonhuman primates, and it is believed to be the basis for the acquisition of advanced mathematical knowledge (Ansari, 2008; Gilmore et al., 2010).

The present work extends the existing literature in the following ways. First, past research has looked into computational estimation skills mainly of addition involving non-symbolic stimuli of up to about 100. The present work examines estimation of multiplication problems of much larger magnitudes in symbolic notation. It uses the *estimation comparison task* with multi-digit multiplication problems. In this task, which was originally introduced by Rubinstein (1985), a multi-digit multiplication problem is presented together with a reference number and participants are asked to indicate whether their estimated answer for that problem is larger or smaller than the reference number. Second, past research did not provide any information on how participants solved this estimation comparison task. This is the first study that explored strategy use with the estimation comparison task.

Thus, two aspects of performance in this task were explored in the present study. The first aspect was the involvement of the ANS, as indicated by the following two signatures of the ANS (e.g., Feigenson et al., 2004): (1) the fact that it is ratio dependent, with enhanced performance for large ratios and (2) the advantage for processing smaller magnitudes over larger ones. Thus, the extent to which accuracy and response latency are affected by the relative distance (ratio) between the exact answer and the reference number and by the size of the numbers – both the size of the multiplicands and the size of the reference numbers – was examined.

Although these effects are considered landmark findings in the numerical cognition field it is not obvious that they will be found in the present study because the distance and the size effects were most consistently found for numbers in the first decade or first hundred. For comparisons of pairs of 4- or 6-digit numbers no distance effect was found (Poltrock & Schwarz, 1984), thus suggesting that when confronted with a 4-digit number, people hold in mind only a sequence of single digits, with no sense of the holistic magnitude of the multi-digit number. Furthermore, past research asking participants to produce an estimate to multi-digit multiplication problems reported relatively poor accuracy (Siegler & Booth, 2005). These patterns of results might be interpreted as reflecting a human limitation to represent such large magnitudes using the ANS. Alternatively, they might reflect task-specific strategies. The present study using the estimation comparison task with multi-digit numbers might shed light on this issue.

The second aspect of this task explored in the present study was strategy use. Past studies using the estimation production task reported that participants employed different simplification strategies, such as rounding both multiplicands up, both multiplicands down, or one up and one down, and multiplied the rounded numbers with or without post-compensation (e.g., Ganor-Stern & Siegler, 2004; Imbo & LeFevre, 2011; LeFevre et al., 1993; Lemaire & Lecacheur, 2002; Lemaire et al., 2000). Such approximate calculation procedures are at least in part taught in school, and they reflect an algorithmic process. The estimation comparison task might be similarly solved by an approximate calculation strategy. However, participants might base their decisions on a coarse, intuitive sense of magnitude grounded in the ANS, without using any calculation. For example, when the problem is 27×68, participants might respond without any calculation that the exact answer is less than 100,000 or is more than 100.

Evidence for such an intuitive sense for multiplicative numerical relationships was recently reported even for young children prior to any formal schooling in multiplication. McCrink and Spelke (2010) examined the ability of 5- to 7-year-old children to estimate the result of multiplication of numerosities relative to a reference quantity. Their performance was above chance level, and it was affected by the ratio between the correct and proposed answer. The fact that the children in this study did not have any formal schooling in multiplication led the researchers to conclude that their performance relies on a core, intuitive multiplication ability which is based on their ANS. However, it is unclear whether it might be applied for much larger magnitudes.

This paper is composed of two experiments. In both experiments college students were presented with a set of multiplication problems. Each problem was presented with a reference number, and participants had to indicate whether they estimated the answer to this problem to be larger or smaller than the reference number. The size of the reference number relative to the exact answer and their relative distance were orthogonally manipulated. The reference number was either larger than the exact answer or smaller than it. The relative distance was either *close* or *far*. In the *close* condition the ratio between the exact answer and reference number was 1:2 (or 2:1) and in the *far* condition it was 1:5 (or 5:1). The list of problems used in this study is provided in Appendix A.

In Experiment 1, multiplication problems of 2-digit (D) $\times$ 2-digit numbers were used. The extent to which accuracy and speed were affected by the size of the reference number relative to the exact answer, and by the relative distance between them was investigated, together with a preliminary analysis of the strategies used by participants in solving this task. Experiment 2 expanded the picture by manipulating problem size, and including, in addition to 2-digit $\times$ 2-digit numbers, multiplication problems of 1-digit $\times$ 2-digit numbers and 2-digit $\times$ 3-digit numbers. This enabled examining the effect of the problem size and any interaction between problem size and the other reference number characteristics. Most importantly, Experiment 2 also included a systematic analysis of the strategies used in solving this task and the relationship between strategy choice and problem characteristics.

2. Experiment 1

In Experiment 1, a set of 72 2D $\times$ 2D multiplication problems was given to a group of college students. Of the 72 problems, in 18 problems the reference number was larger than the exact answer and far from it (i.e., exact answer $\times$ 5), in 18 problems the reference number was smaller than the exact answer and far from it (i.e., exact / 5), in 18 problems the reference number was larger than the exact answer and close to it (i.e., exact $\times$ 2), and in other 18 problems the reference number was smaller than the exact answer and close to it (i.e., exact / 2).

The experiment was composed of two stages. In the first stage, participants responded to the 72 problems by pressing computer keys. The second stage was intended to gather preliminary information about strategy use. In this stage, a random sample of 15 problems was presented again. After responding to each problem as before, participants were asked to describe how they reached their decision. The goal of the second stage of Experiment 1 was to acquire preliminary information about the strategies used by participants in solving this task, as a first step toward a more systematic investigation of strategy choice which was conducted in Experiment 2.