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Are 1/2 and 0.5 represented in the same way?☆

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ABSTRACT

Adults' processing of unit and decimal fractions was investigated using the numerical comparison task. When unit fractions were compared to integers, the pattern of distance effect found suggests that they were perceived to be on the same mental number line as integers; however, their representation was undifferentiated, as they were perceived to have the same magnitude. This was found both with simultaneous and with sequential presentation. When decimal fractions were compared to integers, the pattern of results suggests that they were also represented on the same mental number line with integers, but their representation was differentiated. Possible explanations for the different patterns found for unit and decimal fractions are discussed. Moreover, compatibility between the magnitude of the whole fraction and that of its components relative to the compared integer affected performance in the case of decimal fractions and unit fractions presented simultaneously, but not in the case of unit fractions presented sequentially. This suggests that sequential processing reduces the components representation of fractions and the whole number bias.

1. Introduction

Although most studies in the field of numerical cognition have focused on the processing of integers, recently researchers have started looking at the processing of fractions. This topic is of special interest since it has been shown that competence with fractions can predict children's achievement in math above and beyond measures of general intelligence, working memory, and arithmetic knowledge with whole numbers (Bailey, Hoard, Nugent, & Geary, 2012; Siegler et al., 2012).

Fractions[1] differ from integers in important ways, as fundamental principles that hold for integers do not hold for fractions. The successor principle – that every natural number has a unique "next" number – and the one-to-one correspondence principle, which is central to the process of counting, do not apply to fractions (Gelman, 2009). The situation is even more complex as knowledge about integers may *contradict* knowledge about fractions due to the inverse relation between the magnitude of the fraction denominator and that of the whole fraction. As a consequence, children often exhibit a whole number bias (Ni & Zhou, 2005) — the application of natural number knowledge in situations where it is not appropriate. For unit fractions, this bias makes it harder to select the fraction 1/2 as larger than 1/8, because 2 is smaller than 8 (e.g., Behr, Harel, Post, & Lesh, 1992; Post, Wachsmuth, Lesh, & Behr, 1985). This whole number bias is viewed as an obstacle for learning fractions, and as being responsible at least in part for the poor performance of children in tasks requiring fraction processing (Hartnett & Gelman, 1998; Ni & Zhou, 2005).

Researchers have raised the issue of whether fractions are mentally represented by their holistic values or only by the values of their components. Note that a representation of the holistic value reflects the real value of the fraction, while the components representation might be seen as reflecting the whole number bias. In addition, the presence of a holistic representation could override that of the components, or they could co-exist to form a hybrid representation, as in the case of two-digit numbers (e.g., Nuerk, Weger, & Willmes, 2001).

The most common task used to address this issue is the numerical comparison task, with the distance effect being used as an important indicator for the nature of the mental representation. The distance effect refers to the finding that speed of numerical comparisons is inversely related to the numerical difference between the numbers (e.g., Moyer & Landauer, 1967). This robust finding was interpreted as reflecting the representation of numbers along a continuum referred to as a mental number line (e.g., Dehaene, 1997). To examine whether fractions are represented holistically or componentially, researchers examined if comparisons between pairs of fractions were mainly affected by the numerical difference between the fraction whole values or their components values.

A few studies have found a distance effect reflecting the difference between the fractions' holistic values using behavioral (Meert, Gregoire, & Noël, 2010; Schneider & Siegler, 2010; Sprute & Temple, 2011) and brain imaging measures (Ischebeck, Schocke, & Delazer, 2009; Jacob & Nieder, 2009). Other studies have shown

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[1] The term *fraction* is used in the present paper to refer to a rational number in the form of A/B. The term *unit fraction* refers to a fraction with 1 as the numerator, (e.g., 1/6). The term *decimal fraction* refers to a fraction whose denominator is a power of 10, and which is expressed in the form of 0.A (e.g., 0.7).

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evidence only for a componential representation when unit fractions (Bonato, Fabbri, Umilta, & Zorzi, 2007) and fractions with the same denominators were compared (Meert, Gregoire, & Noël, 2009). Meert et al. concluded that although in some conditions the holistic values of fractions are activated, the access to these holistic values occurs mainly when strategies that rely on components are made difficult.

The results of Iuculano and Butterworth (2011) using the number line task lead to a similar conclusion. When asked to translate numerical magnitudes into spatial positions on a line, both adults and children showed a pattern suggesting linear representation of both integers and fractions. Thus, the authors argued that when the task does not encourage componential strategies, adults and even children demonstrate holistic representation of fractions.

Another question that was raised by past studies is whether fractions and integers, despite their fundamental differences, are represented on the same mental number line. Note that this question is related although not identical to the holistic/components debate, with the relation being that if represented on the same mental number line with integers, fractions are represented holistically at least to some extent (Ganor-Stern, 2012). The distance effect in *mixed* pairs comparisons, which are composed of a fraction and an integer, provides important information on this matter.

Kallai and Tzelgov (2009) looked at this distance effect at two levels; the first, *the integer distance effect*, was the distance between the fractions and each of the integers. This would be reflected in faster comparisons between 1/2 and 8, for instance, than between 1/2 and 3. The second level, *the fraction distance effect*, was the distance between an integer and each of the fractions. For example, a distance effect at this level would be reflected in faster comparisons of 2 to 1/9 than 2 to 1/3. They found an integer distance effect but not a fraction distance effect, leading them to the idea of a generalized fraction representation, according to which when compared to integers, unit fractions are represented on the mental number line with integers, however, with little differentiation among the magnitudes of the represented unit fractions. A recent study by Ganor-Stern (2012) replicated this pattern of results.

Should such a representation be considered to be a holistic or a components representation? As mentioned earlier, if a fraction is represented on the same continuum with integers, it obviously does not have a components representation. However, since it does not include information about the specific value of each fraction, it might be considered to be a generalized holistic representation.

The goal of the present study was to expand the existing knowledge on fraction processing in two directions. First, the effect of sequential presentation on the processing of unit fractions was examined (Experiment 1). This is important as sequential presentation was found to weaken the components representation of two-digit numbers (e.g., Ganor-Stern, Pinhas, & Tzelgov, 2009; Zhang & Wang, 2005). In addition, sequential presentation encouraged the holistic representation of negative numbers on the same mental number line with positive ones (Ganor-Stern, Pinhas, Kallai, & Tzelgov, 2010). Presenting fractions sequentially is expected to reduce the components representation of fractions, and thus the whole number bias, and perhaps even increase the differentiation between the holistic representations of the different unit fractions when compared to integers.

Furthermore, most studies addressing the representation of numbers denoting quantities smaller than 1 focused on fractions in the form of x/y. However, such quantities are often expressed as decimals. Since little research has been devoted to the investigation of the representation of such fractions, the second goal of the present study was to investigate the representations of decimal fractions (Experiment 2).

Decimal fractions, similar to unit fractions, are complex stimuli both perceptually and verbally, and they are used less often than integers are. Neither unit fractions nor decimal fractions are used for counting. Decimal fractions, however, differ from unit fractions in important ways that make them similar to multi-digit integers. First, for decimal fractions, the magnitude of each of the components is positively related to the holistic magnitude of the number. For unit fractions, the relation between the magnitude of the denominator and that of the whole number is negative. Second, for decimal fractions, the relationship between the components magnitude and the fraction magnitude is linear (e.g., the numerical difference between 0.2 and 0.3 is identical to the one between 0.7 and 0.8), while for unit fractions it is not. For example, the numerical difference between 1/2 and 1/3 is different from the one between 1/7 and 1/8, although the components difference is the same. Moreover, the decimal fractions 0.1–0.9 cover the whole range between 0 and 1. In contrast, the unit fractions 1/2–1/9 cover only the range between 0 and 1/2. Last, decimal fractions are written horizontally, in contrast to unit fractions, which are written vertically.

Most research on the processing of decimal fractions has looked at children. Children experience difficulty when learning decimal fractions due to the misattribution of knowledge on integers to such fractions. When comparing the magnitudes of decimal fractions, children seem to follow a set of implicit incorrect rules (Nesher & Peled, 1986). One such rule is that the number with the more digits after the decimal point is larger (e.g., Nesher & Peled, 1986; Rittle-Johnson, Siegler, & Alibali, 2001). This rule probably accounts for the finding that only 15% of 4th graders understood that 0.1 is larger than 0.01 (Desmet, Gregoire, & Mussolin, 2010).

Another rule is that the number with the larger decimal digit is larger. This leads children to select 7.24 as larger than 7.4.

A recent study by Iuculano and Butterworth (2011) investigated the representation of fractions and decimal fractions of children and adults using the number line task. When required to locate a number on a line, participants exhibited a linear representation of both fraction types. However, accuracy and differed substantially. Reaction time and error rates for fractions were much higher than for decimal fractions. Reaction time and error rates for decimal fractions were similar to those of integers. This was true for both adults and 10 year old children.

Examining the processing of both fraction types in the present study might provide insight on the importance of the different characteristics, those that are shared by both fraction types, and those that distinguish between them.

2. Present study

The present study is composed of two experiments that explored the representations of unit and decimal fractions among college students using the numerical comparison task. Only proper fractions representing quantities smaller than 1 were used. To be able to make direct comparisons, both fraction types were composed of the same components, that is, 0–9, and the integers used were also 0–9. In Experiment 1, which examined the representation of unit fractions, the stimuli set included the integers 0–9 and the unit fractions 1/2–1/9. Both simultaneous and sequential presentations were included to experimentally test the effect of presentation mode, and to replicate past findings that used simultaneous presentation. In the simultaneous presentation two numbers were presented one next to the other, and participants compared their magnitudes. In the sequential presentation, the long sequential procedure used by Ganor-Stern et al. (2010) was used. In this procedure a long sequence of numbers is presented and participants have to compare each number with the one that preceded it. In Experiment 2, decimal fractions and integers were presented simultaneously. The integers 0–9 and the decimal fractions 0.1–0.9 (with intervals of one tenth) were used.

Two issues were of special interest. The first issue was whether the mapping of the different fraction types on the same mental number line with integers was the same or different. To answer this