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When meaningful components interrupt the processing of the whole: The case of fractions ☆

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ABSTRACT

Numerical fractions are composed of a numerator and a denominator that are natural numbers. These components influence processing of the fraction. This study was conducted to test whether eliminating the fractional components would result in the processing of fractions as unique numerical entities. Participants that learned to relate fractional values to arbitrary figures in a training task showed automatic processing of the numerical values of the new figures. The processing of fractions written in regular form improved following training, but did not show automatic processing. The results suggest that eliminating the influence of the fractional components allowed individual fractions to be represented in long-term memory.

1. Introduction

It is widely accepted that numerical knowledge is represented by a relatively independent mental module that specializes in quantity perception or in the acquisition of quantitative knowledge (Feigenson, Dehaene, & Spelke, 2004; Gallistel & Gelman, 1992). Some important questions are, 'what types of numbers can be represented as primitive units (i.e., entities represented as such in long-term memory) in this module?' Do they include only integers? Or can real numbers also be represented as such? Fractions (rational numbers), which are in the focus of this study, are a subset of real numbers. A previous work on fractions by the authors (Kallai & Tzelgov, 2009) found that, unlike the case of negative numbers (Fischer & Rottmann, 2005; Shaki & Petrusic, 2005; Tzelgov, Ganor-Stern, & Maymon-Schreiber, 2009), there is a potential for numbers smaller than one to be represented as primitives in long-term memory. Fractions, as a group, were perceived as smaller than natural numbers. The present work further examines this potential for representing various fractional values in long-term memory.

Gelman (2000) argued that natural numbers are the only type of numbers observed in all cultures and animals. She concluded that the counting numbers, as their name indicates, are the "natural" numbers, whereas other kinds of numbers are later and culture-dependent developments. According to this view, natural numbers are the only potential primitives of numerical cognition, while fractions cannot be represented as such. This approach is supported by various findings showing the difficulty of learning and processing of fractions (Ni & Zhou, 2005; Stafylidou & Vosniadou, 2004).

In contrast, some theories suggested that we are phylogenetically tuned to process real numbers rather than only natural numbers. For instance, Mix, Huttenlocher, and Levine (2002) argued that early representations of quantities are not number-based, but total-amount-based, for both discrete and continuous quantities. The accumulator model (Gallistel & Gelman, 1992; Gallistel, Gelman, & Cordes, 2006) was shown to predict both discrete and continuous quantity estimations (Gibbon, 1977). In their review, Gallistel et al. (2006) argued that the real numbers (as a continuum) are the phylogenetic primitives of numerical knowledge and that the acquisition of language, which is discrete in nature, is responsible for the special status of the natural numbers. Leslie, Gelman, and Gallistel (2008) further suggested the existence of an integer system that is mapped to the continuous accumulator that represents real numbers. In accordance with the role of language, a comparative study by Miura, Okamoto, Vlahovic-Stetic, Kim, and Han (1999) showed better understanding of fractions by Korean children, as compared with US and Croatian

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children, prior to school instruction, and attributed it to the literal Korean concepts for fractions.

Regardless of the question of phylogenetic ability to represent fractional values, theories of general learning suggest that mental entities can emerge with experience. For instance, according to the self-organized consciousness (SOC) framework, proposed by Perruchet and Vinter (2002), any 'chunk' of any subject matter (including fractions) can become a mental "primitive" given an adequate amount of practice. (This in turn, raises the question of whether acquired concepts of numbers would become *numerical* primitives, that is, primitive units of the numerical independent mental module.)

Evidence for the spontaneous understanding of fractions by children is mixed. McCrink and Wynn (2007) showed that even six-month-old infants can discriminate between ratios of 1:2 and 1:4 (but not between ratios of 1:2 and 1:3). This ability is similar to infants' ability to discriminate between quantities (Xu & Spelke, 2000), which suggests a similar mechanism for the representation of ratios and quantities. Mix, Levine, and Huttenlocher (1999) argued that very young children (4–6 years old) were somewhat successful in addition and subtraction of fractions, however, the legitimacy of this conclusion was questioned on account of alternative explanations (e.g., Gelman, 2000). Bialystok and Codd (2000) also concluded that children do not understand fractions spontaneously, but rather need special instructions. In formal education, fractions are usually learned by children only after natural numbers are strongly established. Ni and Zhou (2005) argued that this arrangement caused a "whole number bias", which is "the tendency in children to use the single-unit counting scheme applied to whole numbers to interpret instructional data on fractions" (p. 27). There is much evidence for the difficulty experienced by children to understand the concept of fractions (Bright, Behr, Post, & Wachsmuth, 1988; Butterworth, 2007; Hartnett & Gelman, 1998; Shipley & Shepperson, 1990; Siegler, Thompson, & Schneider, 2011; Smith, Solomon, & Carey, 2005). For instance, in a study examining 6th and 8th graders, Siegler et al. (2011) reported low accuracy rates, long reaction time (RT), and erroneous strategies in tasks involving fractions. The authors acknowledged that fractions number line estimation is far from automatic; rather, it appears to be a controlled, strategic process.

The processing of fractions received great attention recently, when researchers debated on the question of whether fractions are being processed by adults holistically or on the basis of their components. Bonato, Fabbri, Umiltà, and Zorzi (2007) asked psychology and engineering students to compare fractions to a standard number of 1/5 or 1. They found that both the distance effect – the increase in RT as the distance between compared numbers decreases (Moyer & Landauer, 1967) – and the SNARC effect – faster responses for small numbers with a left key-press as opposed to faster responses for large numbers with a right key-press (Dehaene, Bossini, & Giraux, 1993)[1] – were determined by the components of the fraction and not by the fraction as a unique unit. The authors concluded that the participants used strategies that were applied on the components of the fractions and that the mental magnitude of the holistic fraction was not accessed.

The results of Bonato et al. (2007) study could have been a consequence of the stimuli set they used, which could have encouraged the use of component-based strategies. Meert, Grégoire, and Noël (2009, 2010) argued that when intentional strategies are made difficult, then the value of the fraction is accessed. Using a comparison task for fractions that shared either their numerators or their denominators or had no common components at all they showed, like Bonato et al. (2007), a distance effect between the components of pairs of fractions that shared the same denominator. However, when fractions differed in their denominators (Meert et al., 2009) or shared no common component (Meert et al., 2010), a higher correlation was found with distance between the values of the fractions than with distance between the components. Meert et al. concluded that although in some conditions the holistic values of fractions are used, the access to these holistic values is not automatic since it is done only when strategies that rely on components are made difficult. Instead, they argue that the mental magnitude of each component is activated and then an operation that approximates the ratio is applied on these magnitudes. Similar results were found by Schneider and Siegler (2010) which also included pairs of fractions that share neither their numerators nor their denominators. The authors concluded that participants accessed holistic representations of fractions; however, slow RT suggested that participants intentionally calculated the ratio between the numerator and the denominator and did not relate to the fraction as one entity representing a certain magnitude. As suggested by Meert et al. (2009, 2010), this study only showed that, when needed, participants of different educational abilities can calculate the ratio of two single- as well as double-digit numbers.

Common to the studies described above is their employment of intentional tasks. In intentional tasks, participants intentionally process the critical dimension of the stimuli (e.g., the numerical value of fractions) and might use different strategies to meet the requirement of the requested task. In automatic tasks, on the other hand, processing of the critical dimension of the stimuli is done without deliberate monitoring (Bargh, 1992, 1997; Tzelgov, 1997). We believe that an effective way to identify the primitives of the numerical system is to recognize the types of numbers that can be processed automatically. Automatic processing of fractions was tested in a previous study by the authors (Kallai & Tzelgov, 2009). In this study, we used a Stroop-like numerical task to test the Size Congruency Effect (SiCE). In Stroop-like phenomena (Stroop, 1935), the influence of a task-irrelevant dimension on the intentionally processed dimension of the stimulus is used to diagnose the processing of the irrelevant dimension as automatic. Henik and Tzelgov (1982) have shown automatic processing of the numerical values of the natural numbers 1–9 by presenting pairs of integers that differ in their numerical and physical sizes, and asking their participants to select the physically larger/smaller integer, while ignoring its numerical value. Slower RT in the incongruent condition (e.g., 3 5) compared with the congruent condition (e.g., 3 5) indicated that the numerical values of the integers were processed even though this processing was ineffective to the task intentionally performed. Kallai and Tzelgov (2009) showed that comparing pairs of fractions using unit fractions (i.e., 1/x) produced a SiCE to the denominators but not to the fractions as holistic entities. In an experiment in which both numerators and denominators vary (similar to the experiment reported by Meert et al., 2009), no SiCE was found. Since the larger the numerator the larger the fraction, but the larger the denominator the smaller the fraction, the lack of SiCE suggested, again, that the components of the fractions and not the holistic fractions were processed under automatic conditions. However, comparing fractions with natural numbers showed automatic processing of fractions as smaller than natural numbers, regardless of the value of their components. This result suggested the existence of a primitive representation of a fraction as an entity "smaller than one". We referred to this entity as a 'generalized fraction' (GeF) and assumed it was based on the general structure of the fraction (i.e., a ratio of two integers). Numerical effects (SiCE and distance effect), found in both intentional and automatic comparisons tasks, suggested that the GeF was processed as a numerical entity. However, while a generalized fraction can be defined by the (maybe perceptual) rule $X/Y<1$, individual fractions have unique meanings of quantity and are represented in specific locations on the number line. Kallai and Tzelgov's results showed that, under automatic conditions, individual fractional values were not processed. Instead, the components of the fractions were processed together with the notion of a GeF.

Imaging techniques were also used to find whether mental representations of fractions exist independently of their components. Recent

[1] The SNARC effect was not found for Hebrew speakers (Shaki, Fischer, & Petrusic, 2009), and therefore was not tested in this study.