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# Integrating natural risks into silvicultural decision models: A survival function approach

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ABSTRACT

In the context of climate change, the frequency and intensity of natural disturbances of silvicultural production, such as storms and insects, are expected to increase. Hence, now more than ever before such factors must be considered in forest management. As a contribution to this topic, this article presents a calculation model implemented in Excel frames, which supports decisions in forest production under changing conditions. Risk is integrated into the model by the Weibull function, which serves as an age-dependent survival function. In order to facilitate an intuitive interpretation of its coefficients, it was used in a reparametrised form. Furthermore, salvage price reductions and cost additions caused by calamities are considered. The target variable is the 'annuity under risk'.

We demonstrate exemplarily how different parameters of the survival function influence the probability distribution and thus the expected value of the annuity of a spruce stand. The differences between the annuities with and without a consideration of risk are interpreted as current, annual risk costs. It can be shown that risk lowers the annuity, whereas scenarios with high risks in the young stand stages have a higher impact than those with high risks in mature stands. In the latter case, adaptation is possible by shortening the rotation period. This does not hold in the case of early risks, which cannot be avoided. For this case, an extension of the rotation length is recommended.

By changing the parameters of the survival function, this scheme allows forest managers to incorporate changing risks into their management planning.



## 1. Introduction and problem outline

Natural disturbances caused by storms, droughts and insects, are — to different extents — an integral part of forest ecosystems (Otto, 1994, pp. 322). To forest managers, however, they constitute considerable risks because they interfere with scheduled operating procedures and objectives, cause additional costs for salvage-harvesting and replanting and decrease revenues from timber sales. As a result of climate change, for central Europe a considerable increase of risks is expected, which is especially caused by lower levels of precipitation during the vegetation period and more frequent droughts and storms (Federal Environment Agency, 2008). Most notably since the 1980s, scientific publications dealing with the causes and the scope as well as with the economic implications of natural hazards in forest management were correspondingly numerous.

Accordingly, in their review articles, Brumelle et al. (1990) and Newman (2002) refer to publications that address the integration of risks into decision models by means of explicit computations of probability distributions of payoffs as a function of activities. Bongiorno (2001), for example, describes the stochastically influenced development of a forest using a Markov decision process (MDP) model, which implicates discrete transition probabilities. The objective is to determine the best decision policy, which maximises the soil expectation value. Numerical solutions for this purpose are found using either successive approximations or linear programming. Kuboyama and Oka (2000) analysed long term data on climate-induced forest damages based on the 'Statistical Yearbook of National Forest Insurance' of Japan to derive empirical, age-class dependent damage probabilities. Using these probabilities they determined the optimal rotation age by means of Monte Carlo simulations (Metropolis and Ulam, 1949). The same approach was used by Dieter (2001), who calculated risk-influenced soil expectation values for beech and spruce in southern Germany. However, risks were described by means of survival functions, which model the chronological sequence of survival probabilities depending on tree species and site conditions. Similar approaches were pursued by others, including Knoke and Wurm (2006) and Beinhofer (2007). On the huge body of literature on forest economics under risk see, e.g., Brumelle et al. (1990) and Newman (2002).

Considerations of these findings in the practice of forestry were proposed, e.g., by Kurth et al. (1987), König (1999) and Kohnle et al. (2008). However, in spite of the fact, that natural risks have a large

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economic impact on forest management, namely on the selection of tree species, thinning practices and rotation cycles, standardised forms of quantification of natural risks and risk management systems have yet to be established in Germany (Gadow, 2000; Gautschi, 2003). In 1990 even Brumelle et al. stated that in practice the explicit use of stochastic approaches in decision-making is rare.

This is the essential starting point of the present article, which is the elaborated version of the paper of Möhring et al. (2010) with a more detailed description of the methodology. One objective is to develop an applicable method for quantifying the calamity-influenced survival probabilities of forest stands. Therefore, the authors use the so-called survival function, the theoretical fundamentals of which are described briefly in the context of survival analysis.[1] In this article, the survival function follows the Weibull distribution, but it was used in a reparametrised form (see Staupendahl, 2011) that makes it easy to interpret its coefficients. Furthermore, a method is shown that allows the immediate calculation of the 'annuity under risk', which supersedes the application of iterative or numerical methods like the Monte Carlo technique or linear programming. Finally, by means of calculations with different survival functions, it is shown how the costs of risk and the risk-adjusted optimal rotation age can be determined. This approach, the authors hope, will contribute to the enhancement of decision-making in forestry (see also Deegen, 1994) and will promote the integration of risks into practical forest planning and evaluation.

## 2. The survival function

Whenever the economic impact of natural risks in forestry is to be quantified, knowledge of the probability that a given stand at a specific site reaches or even exceeds a specific age is required. The probability of survival can alternatively be interpreted as the share of the afforested area that, on average, is still present at a specific age.

### 2.1. Fundamentals about the survival function

The probability of survival is a central term of survival analysis, which investigates the distribution of the non-negative random variable $T$, whereas $T$ describes the point of time in which an event of interest occurs (a special realisation of $T$ is denoted by $t$). Here, $T$ is defined as the 'stand age at the time of a calamity-induced dropout'. According to Klein and Moeschberger (1997, pp. 21), the pattern of $T$ can usually be characterised by the following four functions.

The probability density function $f(t)$ describes the frequency distribution of the points in time, in which the event occurs. In the case of approximate continuously measured time, it is defined as

$$f(t) = \lim_{\Delta t \to 0} \frac{P(t \le T < t + \Delta t)}{\Delta t}, \text{with } t \ge 0. \tag{1}$$

For small $\Delta t$, $f(t)\Delta t$ may be thought of as the approximate unconditional probability that the dropout will occur at time $t$. The cumulative distribution function $F(t)$, as the integral of the density function, gives the probability that a drop-out has occurred by time $t$:

$$F(t) = P(T \le t) = \int_0^t f(x)dx. \tag{2}$$

The survival function $S(t)$ is the complement of $F(t)$. Thus, it gives the probability that a stand survives at least time $t$:

$$S(t) = P(T > t) = 1 - F(t). \tag{3}$$

The probability that the event occurs at a certain time, conditional on the subject having survived to that time, is also important. The density of this conditional drop-out probability is called the risk or hazard function (hazard rate) $h(t)$, defined as

$$h(t) = \lim_{\Delta t \to 0} \frac{P(t \le T < t + \Delta t | T \ge t)}{\Delta t} = \frac{f(t)}{S(t)}. \tag{4}$$

The hazard rate is a non-negative function and describes the instantaneous tendency to a state change, given that a state change did not occur previously (Ludwig-Mayerhofer, 2009). For small $\Delta t$, $h(t)\Delta t$ gives an approximation of the conditional drop-out probability. The hazard function characterises the risk of failure depending on the survival time. For ageing systems, its value increases with increasing age; the longer such a system survives, the higher its risk of failure is.

In the case of discrete time, which is normally considered in forest practice, the unconditional probability that a newly established forest stand drops out in age class $i$, is given by

$$f(t_i) = P(T = t_i) = S(t_{i-1}) - S(t_i), \tag{5}$$

where $t_i$ denotes the average of age class $i$ and $S(t_i)$ gives the discrete survival function, with $i = 1, 2, \ldots, t_0 = 0$ and $S(t_0) = 1$. Accordingly, the time discrete hazard rate (Fahrmeir, 2007, pp. 26) gives the conditional probability that a stand, which survived until age class $i$, drops out in this age class:

$$h(t_i) = P(T = t_i | T \ge t_i) = \frac{f(t_i)}{S(t_{i-1})} \tag{6}$$

In other words, $h(t_i)$ indicates the fraction of the area in age class $i - 1$, which is dropped out on average in age class $i$. Thus, $h(t_i)$ is the complementary value to the transitional probability $p(t_i)$, which was introduced to forestry by Suzuki (1971, 1983) and is widely used in the literature (e.g., Möhring, 1986; Kurth et al., 1987; Deegen, 1994). The discrete survival probability $S(t_i)$ can be calculated as the product of these transitional probabilities:

$$S(t_i) \prod_{j=1}^{i} p(t_i), \text{with } p(t_i) = 1 - h(t_i). \tag{7}$$

Several distribution types are available to give the parametric description of $T$, including the exponential, log-normal or log-logistic distributions. Here, according to Pienaar and Shiver (1981), Kouba (2002) and Holecy and Hanewinkel (2006), the Weibull distribution (Weibull, 1951) was chosen because it requires only two parameters while at the same time being quite flexible. If $T$ is Weibull distributed with scale parameter $\beta$ and shape parameter $\alpha$, the density, distribution, survival and hazard function are given as follows, with $t > 0$ (Klein and Moeschberger, 1997, p. 37)[2]:

$$f(t) = \frac{a}{\beta} \cdot \left(\frac{t}{\beta}\right)^{a-1} \cdot \exp\left[-\left(\frac{t}{\beta}\right)^{a}\right] \tag{8}$$

$$F(t) = 1 - \exp\left[-\left(\frac{t}{\beta}\right)^{a}\right] \tag{9}$$

$$S(t) = \exp\left[-\left(\frac{t}{\beta}\right)^{a}\right] \tag{10}$$

[1] Survival analysis is most often defined as a class of statistical methods for studying the occurrence and timing of events — most often death (e.g., Cox and Oakes, 1984).

[2] However, an alternative notation is used in this paper, whereas $\lambda$, given by Klein and Moeschberger (1997, p. 37) equals to $\beta^{-\alpha}$.