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Navigation as a source of geometric knowledge: Young children's use of length, angle, distance, and direction in a reorientation task

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ABSTRACT

Geometry is one of the highest achievements of our species, but its foundations are obscure. Consistent with longstanding suggestions that geometrical knowledge is rooted in processes guiding navigation, the present study examines potential sources of geometrical knowledge in the navigation processes by which young children establish their sense of orientation. Past research reveals that children reorient both by the shape of the surface layout and the shapes of distinctive landmarks, but it fails to clarify what shape properties children use. The present study explores 2-year-old children's sensitivity to angle, length, distance and direction by testing disoriented children's search in a variety of fragmented rhombic and rectangular environments. Children reoriented themselves in accord with surface distances and directions, but they failed to use surface lengths or corner angles either for directional reorientation or as local landmarks. Thus, navigating children navigate by some but not all of the abstract properties captured by formal Euclidean geometry. While navigation systems may contribute to children's developing geometric understanding, they likely are not the sole source of abstract geometric intuitions.

1. Introduction

Human adults can conceive of lines so thin that they have no thickness, so long that they never end, and so perfectly parallel that they never meet. The nature and development of these intuitions of Euclidean geometry have long fascinated philosophers and scientists, because they are both so clear and so elusive. Geometrical intuitions are robust enough to support the development of a vast edifice of formal mathematics, and they underlie a host of cultural achievements from measurement to engineering to the visual arts. Moreover, geometrical intuitions may be universal across human cultures, despite the culturally variable ways in which geometry is used (Dehaene, Izard, Pica, & Spelke, 2006; Izard, Pica, Spelke, & Dehaene, 2011; see also Plato, ca. 380, B.C./1949). Nevertheless, the points and lines of Euclidean geometry, and the axioms and theorems that relate them, elude our direct perceptual experience. Where do these geometric intuitions come from, and how do children become sensitive to the relations that they capture?

The present research is guided by an old idea: sensitivity to the fundamental relations of Euclidean geometry arises from the systems that guide children's navigation. Although the earth is round and the local terrain is bumpy, navigation over short distances can be captured by the fundamental properties of Euclidean plane geometry including *length* (the lengths of individual surfaces and objects), *angle* (the relative orientations of two surfaces or edges with respect to one another and the size of the corner that they form when conjoined), *distance* (the displacement of a surface or object from other objects or from one's current

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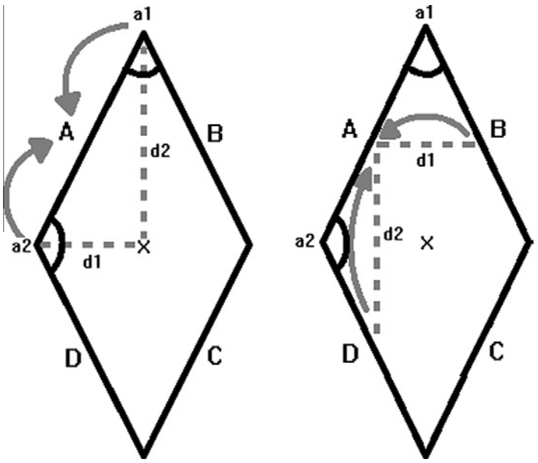

Fig. 1. Illustrations of possible ways in which distance, angle, and direction can be used for reorientation in a rhombic environment in either egocentric (left) or allocentric (right) coordinates. Given four locations (A, B, C, D) that are at the center of each wall and therefore equidistant from the center of the room (array tested in Experiment 2), the left diagram shows distance differences from the center position from which location A (and its geometric twin C), for instance, can be encoded using distance and direction (i.e., "left of the farthest point of the surrounding layout ($d2$)" or "right of the closest point of the surrounding layout ($d1$)"). The right diagram shows distance differences between the surfaces themselves from which A (or C) can be encoded (i.e., "left of the narrower space between surfaces ($d1$)" or "right of the wider space between surfaces ($d2$)") Note that distances could be measured either from the centers of walls or as the average of the distances of each point on a wall. Note that in principle, measurements of angle sizes could substitute for measurement of distances, in either reference frame (e.g., "left of the narrower angle," or "right of the wider angle").

station point) and *sense* or *direction* (the relative positions of surfaces or objects with respect to other objects or to one's own facing direction: Fig. 1).

How do geometric measurements of length, angle, distance and direction guide navigation? It has long been theorized that these measurements apply primarily to the internal encoding of proprioceptive cues to track one's movement, because movements of the eyes, head and body can be related to one another through the axioms and postulates of Euclidean geometry (Descartes, 1637/2001). According to the cognitive map hypothesis, as animals navigate from place to place, they form an allocentric internal representation of the environment that preserves the geometric relationships between the various landmarks and locations within it, based on the distances and directions that they travel (Gallistel, 1990; O'Keefe and Nadel, 1978; Tolman, 1948).

Despite the attractiveness of the view, a wealth of evidence casts doubt on the thesis that geometrical representations and computations underlie navigators' representations of the paths on which they travel. Although inertial navigation, or *path integration*, has been demonstrated in many different animals, from desert ants (Wehner & Srinivasan, 1981) to dogs (Chapuis & Varlet, 1987) to humans (Landau, Gleitman, & Spelke, 1981; Loomis, Klatzky, Golledge, & Philbeck, 1999), experiments cast doubt on the thesis that any navigating animals use path integration to build a Euclidean geometric map of the environment. Both insects (Wehner & Wehner, 1990) and humans (Foo, Warren, Duchon & Tarr, 2005) show striking limitations in their abilities to use distance, angular, and sense relations between paths navigated. Instead, a better candidate source of geometrical knowledge comes from studies of navigating animals' sensitivity to the structure of the surrounding surface layout.

"Geometry" means "the measurement of the earth." Several decades of research in spatial navigation reveal that animals, including human toddlers, use geometric properties of their environment to guide their navigation to goal locations (e.g., Cheng & Newcombe, 2005; Gallistel, 1990). Because navigating creatures can ignore all features of the terrain over short distances when they are oriented, and instead find their way by using path integration to update their position, animals' sensitivity to the geometry of the external environment is best revealed when they are disoriented. When an animal loses its own sense of direction, it must use external directional cues to reorient – to regain its heading and position with respect to locations in the environment. For example, when rats watch as food is buried in one corner of a rectangular arena and then are disoriented by covered rotation and placed back in the arena, they reorient themselves primarily according to the shape of the room, and therefore search with equal frequency at the correct corner and its diagonally opposite geometric twin (Cheng, 1986). The rats' surprising failure spontaneously to use other available cues (such as distinctive odors or wall patterns) to break the room's symmetry led to the formulation of the *geometric module* hypothesis. According to this hypothesis, disoriented animals regain their heading by establishing the congruence between the shape of the room as it is currently perceived and a representation of the room as it was previously experienced from a specific, known direction. While there is still an ongoing debate regarding whether these representations are egocentric and viewpoint-dependent (e.g., Wang & Spelke, 2002) or allocentric and viewpoint-independent (e.g., Burgess, 2006), many species of animals, from ants to chicks to human toddlers, have been shown to navigate spontaneously by the shape of the surrounding environment after they are disoriented (Brown, Spetch, & Hurd, 2007; Cheng & Newcombe, 2005; Chiandetti & Vallortigara, 2008; Sovrano, Bisazza, & Vallortigara, 2002; Wystrach & Beugnon, 2009), consistent with the hypothesis that a computation of the geometric shape of the environment underlies reorientation (Cheng, 1986; Cheng & Gallistel, 1984).

The geometric reorientation hypothesis has recently been challenged by image-matching theories of navigation that question the necessity for any abstract geometric content in the representations underlying reorientation.[1] According to image-matching theories, animals take visual

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[1] A different aspect of Cheng and Gallistel's original hypothesis, that reorientation depends on an encapsulated system sensitive only to surface layout geometry, has also been challenged (e.g., Newcombe & Ratliff, 2007; Sovrano, Bisazza, & Vallortigara, 2003). We do not discuss Cheng and Gallistel's claims concerning the architecture of navigation systems in this article, but only their claims concerning the geometric content represented by these systems.