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Brief article

Linear mapping of numbers onto space requires attention

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ABSTRACT

Mapping of number onto space is fundamental to mathematics and measurement. Previous research suggests that while typical adults with mathematical schooling map numbers veridically onto a linear scale, pre-school children and adults without formal mathematics training, as well as individuals with dyscalculia, show strong compressive, logarithmic-like non-linearities when mapping both symbolic and non-symbolic numbers onto the numberline. Here we show that the use of the linear scale is dependent on attentional resources. We asked typical adults to position clouds of dots on a numberline of various lengths. In agreement with previous research, they did so veridically under normal conditions, but when asked to perform a concurrent attentionally-demanding conjunction task, the mapping followed a compressive, non-linear function. We model the non-linearity both by the commonly assumed logarithmic transform, and also with a Bayesian model of central tendency. These results suggest that veridical representation numerosity requires attentional mechanisms.

1. Introduction

Most adult humans can estimate the numerosity of a group of items, as can infants (Xu, Spelke, & Goddard, 2005) – including newborns (Izard, Sann, Spelke, & Streri, 2009) – and many non-human animals, including primates, parrots and even fish (Agrillo, Dadda, Serena, & Bisazza, 2009; Gallistel & Gelman, 1992; Nieder, 2005; Pepperberg, 2006). Numerosity shares many properties with other perceptual attributes, such as obedience of Weber's law (Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004; Ross, 2003) and susceptibility to adaptation (Burr, Anobile, & Turi, 2011; Burr & Ross, 2008). Importantly, the ability to discriminate number, which improves during development (Halberda & Feigenson, 2008; Piazza et al., 2010), is strongly predictive of future mathematical ability (Halberda, Mazzocco, & Feigenson, 2008).

Number and space are intrinsically interconnected. Mapping of numbers onto space plays a fundamental role for many aspects of mathematics, including geometry, Cartesian coordinates and mapping real and complex numbers onto lines or planes (Butterworth, 1999; Dehaene, 1997). Recent work has shown that children's conceptions of how numbers map onto space shifts radically during the early school years (Booth & Siegler, 2006; Siegler & Booth, 2004; Siegler & Opfer, 2003). Kindergarten children can represent numbers in space in a non-random manner, but their representation is compressed, seemingly logarithmic (placing, for example, the number 10 near the midpoint of a 1–100 scale). The compressive non-linearity becomes progressively more linear over the first 3 or 4 years of schooling (Booth & Siegler, 2006; Siegler & Booth, 2004; Siegler & Opfer, 2003), leading some to suggest that the "native" system of representating numbers may be logarithmic, which becomes linearized by schooling (Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010). Strong support for this idea comes from a recent study of the Mundurucu, an Amazonian indigenous group with a limited number lexicon and little or no formal training:

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both adults and children of this tribe map numbers and numerical quantities onto space in a logarithmic fashion (Dehaene, Izard, Spelke, & Pica, 2008). This points to both genetic and cultural roots to numerical mapping: the ability to represent numbers in space appears to be innate, but formal mathematical training is required to refine the representation from logarithmic to linear. Interestingly, dyscalculic children from developed societies also show a more logarithmic representation of the numberline than controls (Ashkenazi & Henik, 2010; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Geary, Hoard, Nugent, & Byrd-Craven, 2008).

However, the notion that number is encoded in a true logarithmic fashion has been challenged from many fronts (Gallistel & Gelman, 1992; Karolis, Iuculano, & Butterworth, 2011). Alternate explanations have also been put forward for the non-linearities in the numberline representation, such as proportion judgments relative to the ends and centers of the numberline (Barth & Paladino, 2011). Another possibility, which we advance in this study, is that the non-linearity is an example of the well known "central tendency of judgment", reported by Hollingworth back in 1910: "judgments of time, weight, force, brightness, extent of movement, length, area, size of angles all show the same tendency to gravitate toward a mean magnitude" (Hollingworth, 1910). This old concept has recently been relaunched in the context of Bayesian analysis to model interval reproduction judgments (Cicchini, Arrighi, Cecchetti, Giusti, & Burr, 2011; Jazayeri & Shadlen, 2010).

Attention has been shown to play an important role in number perception. Attentional-training (through videogame playing) increases the subitizing range (Green & Bavelier, 2003). Although subitizing has often been considered to be "pre-attentive", several studies have shown that it in fact highly attentional-dependent, suffering considerably when attention is diverted with dual-task or attentional-blink paradigms (Burr, Turi, & Anobile, 2010; Railo, Koivisto, Revonsuo, & Hannula, 2008; Vetter, Butterworth, & Bahrami, 2008; Xu & Liu, 2008). Under dual-task attentional conditions, number discrimination (measured by Weber fraction) in the subitizing range falls to the same level as the estimation range (Burr et al., 2010). Attention also affects adaptation to numerosity (Burr et al., 2011).

In this study we ask whether spatial mapping of numbers depends on attention. Adult observers positioned dot-stimuli on a numberline, with and without a concurrent demanding color-conjunction task. With the attention-demanding task, the spatial representation of number, linear under normal viewing, shows clear non-linear compression. One interpretation of the results is that the native system of number representation is logarithmic, even in typical adults with normal mathematical ability, and that the linearization of this representation requires attention. However, we also explore the possibility that the compressive non-linearity results from a central tendency like that described by Hollingworth (1910) for many sensory judgments, which we model quantitatively within the Bayesian context.

2. Methods

2.1. Participants

Four subjects with normal or corrected-to-normal vision participated in this study, one author and three naïve to the goals of the study. All subjects were graduate students, two with previous experience in numerosity judgment tasks (three female, one male; mean age 26).

2.2. Stimuli and procedure

The stimuli were generated and presented under Matlab 7.6 using PsychToolbox routines (Brainard, 1997). They were displayed in a dimly lit room on a 13-in. Macintosh monitor with 1440 × 900 resolution at 60 Hz refresh rate, mean luminance 60 cd/m^2, viewed binocularly from 57 cm. The stimulus sequence is illustrated in Fig. 1a. Each

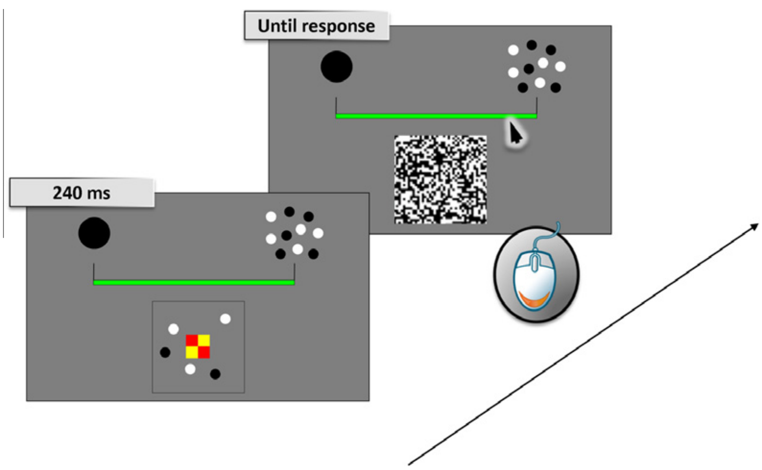

Fig. 1. Illustration of stimuli sequence. At onset of each trial observers view the number line, marked at each end with a single dot to the left and 10, 30 or 100 dots to the right. On key press, the dot stimulus appears, together with four colored squares in the center of the dot cloud. After 240 ms a binary pixel random-noise mask was displayed until subjects respond. Subjects respond first to the color conjunction task (in the dual-task condition, otherwise they ignore it), then mouse-click the numberline at the position they think the dot cloud should occupy.