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Spearman's law of diminishing returns in hierarchical models of intelligence for children and adolescents

Matthew R. Reynolds *, Timothy Z. Keith

The University of Texas at Austin, Department of Educational Psychology, 1 University Station D5800, Austin, TX 78712–0383, United States

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Abstract

Spearman's "law of diminishing returns" or SLODR refers to a decrease in *g* saturation as ability level increases. SLODR has been demonstrated in a number of intellectual batteries but several important aspects of the phenomenon are not yet well understood. We investigated the presence of SLODR in the Kaufman Assessment Battery for Children—Second Edition (KABC-II), a popular measure of intelligence for children. We used confirmatory factor analysis to investigate the invariance of two hierarchical factor structures across ability groups; the subtest variance explained by the ability factors across groups; and whether SLODR was produced only by subtests with low loadings on the general ability factor. We found that SLODR was present in the KABC-II, and its presence was not dependent on the hierarchical model of intelligence. Moreover, our findings suggest that SLODR acts on *g* and not on the broad abilities, although the contribution of *g* to various broad abilities is lower in the high ability group. Finally, SLODR was not produced by the subtests with the lowest *g* loadings on the general factor.

Keywords: Spearman's law of diminishing returns; Intelligence; Confirmatory factor analysis; Higher-order models; Nested factor models

1. Introduction

Spearman (1927) first observed that intercorrelations among cognitive ability tests (i.e., *g* saturation) were higher in low ability groups compared to high ability groups. He referred to this phenomenon as the "law of diminishing returns." Over the past fifteen years, studies of various intelligence test batteries have supported Spearman's law of diminishing returns (SLODR) (e.g., Deary et al., 1996; Detterman & Daniels, 1989; Evans, 1999; Jensen, 2003; Legree, Pifer, & Grafton, 1996). Although a few studies have not found support for SLODR (e.g., Fogarty & Stankov, 1995; Hartmann & Reuter, 2006; Hartmann & Teasdale, 2004) the presence of SLODR in intelligence test batteries is generally supported.

1.1. SLODR and broad abilities

Several aspects of SLODR remain less well understood. SLODR implies that at higher levels of *g*, *g* becomes less important relative to other abilities and skills that a person has (Jensen, 1998). If so, then *g* should account for less of the variance in subtest scores for those of high, as opposed to low, ability. Studies of SLODR typically investigate changes in the *g* factor while tending to ignore potential changes in other

* Corresponding author. Tel.: +1 512 917 5184; fax: +1 512 475 7641.
E-mail address: matthew.reynolds@mail.utexas.edu (M.R. Reynolds).

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abilities. One unanswered question in such research is what happens to the diminishing variance accounted for by *g* with increasing levels of ability? Does the variance explained by various broad abilities correspondingly increase with ability? To understand the nature of SLODR, the effects of SLODR on *g* should be studied simultaneously with possible effects on other broad abilities, such as crystallized intelligence (Gc), fluid reasoning (Gf), and visual processing (Gv).

One study that did examine the variance explained by broad abilities was conducted by Carlstedt (2001), who studied whether the variance contributed by the broad abilities increased at higher levels of *g*. Confirmatory factor analysis (CFA) was used to decompose the variance accounted for by crystallized intelligence and spatial visualization. Both abilities explained more variance at higher levels of *g*, suggesting that broad abilities become more important as *g* levels increase. Carlstedt compared multiple ability groups, a strategy that allowed for an assessment of the progressive nature of the variance accounted for by these abilities. This strategy also necessarily decreased the general factor variance to negligible levels, thus precluding the simultaneous test of the influence of both *g* and the broad abilities. We seek to expand on those findings by including a *g* factor in our analyses so that the variance accounted for by *g* is evaluated simultaneously with the variance accounted for by other factors.

1.2. SLODR and g loadings

As noted by Jensen (2003), it was generally assumed that SLODR acted indiscriminately on various subtests of an intelligence battery. Contrary to this assumption, Jensen (2003) showed that SLODR was produced primarily by subtests with low *g* loadings in the Wechsler Intelligence Scale for Children-Revised (Wechsler, 1974) and in the Wechsler Adult Intelligence Scale (Wechsler, 1981). This finding suggests important information about the nature of SLODR, but requires replication. Another purpose of this research is to test whether SLODR is produced only, or primarily, by subtests with low *g* loadings.

1.3. Confirmatory factor models for studying SLODR

Most studies of SLODR have employed principal component analysis or exploratory factor analysis as the main method for analysis, typically examining the first principal component or factor as an estimate of *g*. This approach does not recognize abilities other than *g* as possible influences of cognitive performance on cognitive ability tests. For example, on tests that require people to recite words or numbers in order, short-term memory is important, or on tests that require putting together pieces of triangles to form patterns, visual–spatial ability influences performance above and beyond the general factor. Factor analytic studies of human cognitive abilities need to use models that reflect the hierarchical and multi-factorial structure of human cognitive abilities including models that allow for the effects of *g* and other important cognitive abilities (Keith, 2005). Following this logic, the present study used hierarchical CFA to examine the presence of SLODR in a test designed to measure multiple, hierarchical abilities. CFA offers several advantages for the study of SLODR, one of which is that it allows for the study of simultaneous effects of *g* and other broad abilities.

Another advantage of CFA is that it also allows for a comprehensive and flexible testing of factorial invariance. At its most basic level, a test of SLODR is a test of factorial invariance across high and low ability groups. If the effect of *g* varies across ability groups, this difference should appear as differences in first-, or (in higher-order models) higher-, order *standardized* factor loadings. In the *unstandardized* solution, SLODR should appear as differences across groups in the variance of the *g* factor. CFA also makes it easy to calculate the proportion of explained variance in the subtests accounted for by the latent factors. The variance can be decomposed to show how the latent constructs account for subtest performance, allowing for a comparison of the variance explained by *g* and the variance explained by the broad abilities.

CFA is especially useful for the analysis of tests that are based on strong underlying theory, or for those whose factor structure is well known. CFA is particularly useful for the present study because the intelligence test battery under investigation, the Kaufman Assessment Battery for Children—Second Edition (KABC-II), is one that has a well-known factor structure aligned with a contemporary theory of intelligence (Carroll's three-stratum theory of intelligence, 1993, 1997). Another advantage of CFA is that it allows for rigorous empirical testing of competing theoretical models of human cognitive abilities. One question not yet resolved regarding the psychometric structure of intelligence is how to model *g* in relation to other ability factors. For example, is *g* better conceptualized as a higher-order factor that influences the first-order factors directly and the manifest variables only through the first-order factors (a higher-order model)? Or alternatively, is *g* modeled better as a factor that shares the same factor