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Estimating the maximum growth rate from microbial growth curves: definition is everything

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Abstract

The maximum growth rate (μ_{max}) is an important parameter in modelling microbial growth under batch conditions. However, there are two definitions of this growth parameter in current use and some of the comparisons of data made in the literature fail to acknowledge this important fact.

We compared values of μ_{max} obtained by applying the Gompertz, logistic and Baranyi–Roberts models to experimental data on the growth of *Listeria monocytogenes* and *Listeria innocua* using both absorbance and viable counts measurements of cell concentration. All three models fitted the experimental data well, however, the values of μ_{max} obtained using the Gompertz and logistic models were similar to each other but substantially different from those predicted by the Baranyi–Roberts model. The latter growth model was used to derive a second estimate of μ_{max} based on the slope at the inflection point of the growth curve function; this value was in closer agreement with those obtained using the Gompertz or logistic models. Conditions were identified when values of μ_{max} based on different definitions would converge towards one another.

Keywords: Predictive microbiology; Microbial growth curve; Growth model

1. Introduction

There is increasing interest in being able to predict the consequences of microbial growth on foods during storage. If methods can be developed to give realistic predictions, considerable savings can be made in the costs associated with laboratory challenge testing of foods (Baranyi and Roberts, 1994).

The time-dependent increase in the microbial population in a closed system is referred to as a growth curve and fundamental to all predictive methods is a requirement to mathematically model, either partly or fully, growth curves for micro-organisms of particular interest over a range of environmental conditions. Numerous expressions have been proposed including some, which as Grijspeerdt and Vanrolleghem (1999) have pointed out, were never envisaged as having been formulated for this purpose. Zwietering et al. (1990) surveyed a number of commonly used empirical expressions—including the Gompertz and logistic expressions—and 're-parameterized' some of them in an attempt to give physical meaning to their parameters. Approaching the problem from a more mechanistic basis, Baranyi and his co-workers (Baranyi et al., 1993; Baranyi and Roberts, 1994; Baranyi, 1997) have proposed and developed a new family of models. The Baranyi–Roberts (1994) model is the most widely used of these.

The Gompertz and logistic models have a similar sigmoid shape with a clear inflection point, whereas the Baranyi model is geometrically different since it shows a quasi-linear segment during the exponential phase.

Selecting the most appropriate growth model is often a matter of trial and error. Moreover, different criteria

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Nomenclature

D	limit of ln VC/VC($t=0$) or limit of ln Abs/Abs($t=0$)
A	integral of adjustment function
h_0	parameter of Baranyi–Roberts model
y_0	ln VC or ln abs at $t=0$
$y_{\max}$	maximum value of ln VC or ln Abs
x	cell concentration
x_0	cell concentration at $t=0$
$x_{\max}$	maximum cell concentration

Greek Symbols

λ	lag phase duration (hours)
μ	growth rate (h^{-1})
$\mu_{\max}$	maximum growth rate (h^{-1})

for determining the suitability of one particular model over another vary: some authors have relied on mathematical measures of goodness of fit (Buchanan et al., 1997), whilst others have focussed on direct comparisons of particular growth parameters as predicted by the various models (Membré et al., 1999). The important growth parameter $\mu_{\max}$, the maximum growth rate, is a case in point. Two definitions for this parameter are in current use. One is based on the inflection of the slope of the growth curve in the exponential phase (Zwietering et al., 1990) whilst the other is taken to be the growth rate at infinite dilution (Baranyi et al., 1993). Confusion may arise if values of $\mu_{\max}$ based on different definitions of the parameter are compared with one another. The situation is further complicated by the fact that different approaches to the gathering of growth data are in current use. Some workers use viable counts as a measure of cell concentration, whilst others use optically based methods.

In this work, we illustrate some of the anomalies that can arise by modelling growth rate data for *Listeria monocytogenes* and *Listeria innocua* using the logistic, Gompertz and Baranyi growth models. Data is obtained for these two organisms both in the form of viable counts and absorbance measurements. *L. monocytogenes* is the cause of a significant number of food-poisoning incidents in developed countries (Farber and Peterkin, 1991). *L. innocua* is not pathogenic but is often isolated alongside the latter and some reports suggest that it can mask the presence of *L. monocytogenes* (Cornu et al., 2002).

2. Materials and methods

2.1. Growth models

In their re-parameterization of empirical expressions applied to growth curves, Zwietering et al. (1990) took $\mu_{\max}$ to be the tangent to the growth curve at its inflection point, and the lag time as the intercept of the tangent at the inflection point with the horizontal tangent at y_0. On this basis the Gompertz expression became

$$y = D\exp\left(-\exp\left(\frac{\mu_{\max}e}{D}(\lambda - t) + 1\right)\right) \tag{1}$$

and the logistic expression

$$y = \frac{D}{1 + \exp((4\mu_{\max}/D)(\lambda - t) + 2)}, \tag{2}$$

where $y = \ln(x/x_0)$.

In the model proposed by Baranyi et al. (1993) the variation of the cell population (x) with time is described by a first-order differential equation

$$\frac{dx}{dt} = \alpha(t)\mu(x)x \tag{3}$$

the function $\alpha(t)$ is called the "adjustment function" and has the following characteristics: it is monotonous increasing; $0 \leqslant \alpha < 1$ and $\lim_{t\to\infty}\alpha(t) = 1$. Moreover, the following relationship for the growth rate is assumed:

$$\mu = \mu_{\max}\left(1 - \frac{x}{x_{\max}}\right). \tag{4}$$

This model can be rewritten in its generic form (Baranyi and Roberts, 1994)

$$\mu(t) = \frac{1}{x(t)}\frac{dx}{dt} = \mu_{\max}\alpha(t)f(t), \tag{5}$$

where $\alpha(t)$ is the previously described adjustment function and $f(t)$ is monotonous decreasing; $f(0) = 1$ and $\lim_{t\to\infty}f(t) = 0$.

Baranyi and his co-workers were able to derive solutions to this differential equation under certain conditions, e.g. fixed temperatures. Initially this was done using six parameters (Baranyi and Roberts, 1994), but this was later reduced to four parameters in Baranyi (1997) as follows:

$$y(t) = y_0 + \mu_{\max}A(t) - \ln\left(1 + \frac{e^{\mu_{\max}A(t)} - 1}{e^{(y_{\max} - y_0)}}\right), \tag{6}$$

where

$$A(t) = t + \frac{1}{\mu_{\max}}\ln(e^{-\mu_{\max}t} + e^{-h_0} - e^{-\mu_{\max}t - h_0}) \tag{7}$$