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Estimation of winter respiration rates and prediction of oxygen regime in a lake using Bayesian inference☆

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Abstract

In this paper, we estimate the winter respiration (oxygen depletion per unit area of hypolimnetic surface) in a hyper-eutrophic shallow lake (Tuusulanjärvi) in the northern hemisphere (Finland, northern Europe, latitude 60°26′, longitude 25°03′) under ice-cover periods in the years 1970–2003. We present a dynamic nonlinear model that can be used for predicting of the oxygen regime in following years and to dimensioning of needed artificial oxygenation efficiency that will prevent fish kills in the lake. We use Bayesian estimation of respiration using Markov chain Monte Carlo (MCMC) method (Adaptive Metropolis–Hastings algorithm). This allows for analysis and predictions that take into account all the uncertainties in the model and the data, pool information from different sources (laboratory experiments and lake data), and to quantify the uncertainties using a full statistical approach. The mean estimated respiration in the study period was $301 \pm 105\,\mathrm{mg\,m^{-2}\,d^{-1}}$, which is on the upper limit of winter respiration of eutrophic Canadian lakes on the same latitude. The reference rate of the respiration k ($\mathrm{d^{-1}}$) at 4 °C indicated cyclic behavior of about 9-year amplitude and had a statistically significant negative trend through out the study period. The temperature coefficient and respiration rate of the model prove to be highly correlated and unidentifiable with the given data. The future winters can be predicted using the posterior information coming from the past observations. As new observations arrive, they are added to the analysis. Methods are shown to be applicable to the dimensioning of artificial oxygenation devices and to the anticipation of the need for oxygenation during the winter.

Keywords: Lake winter respiration; Model uncertainty; Dynamical model; Bayesian modelling; MCMC

1. Introduction

In this research, we model winter respiration in Lake Tuusulanjärvi as total consumption of the oxygen in the lake ($\mathrm{mg\,m^{-2}\,d^{-1}}$), which includes both the consumption in the lake water and on the bottom sediment. Our principle aim is not to develop a complicated oxygen

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model, but to study the changes in the condition of the lake as manifested by yearly average respiration, to predict future oxygen regime in the lake, to dimension probabilistically artificial oxygenation efficiency and to illustrate the multiplicity of model uncertainties and the strengths of the MCMC method in tracking these. The identifiability and uncertainty analysis of large environmental simulation models (Scavia, 1980; Adams VanHarn, 1998; Brun et al., 2001; Omlin et al., 2001a,b; Reichert and Vanrolleghem, 2001) have by now manifested some shortcomings in the scientific basis of complicated ecological model equations and of their parameter ranges. This is why the use of complicated water quality models as quantitative forecasting tools without comprehensive and well-designed measuring campaigns has been challenged and why new interest has been developed in the application of methods that can better trace unidentifiabilities and uncertainties in the models (e.g. MCMC) (Harmon and Challenor, 1996; Adams VanHarn, 1998; Omlin and Reichert, 1999; Annan, 2001; Borsuk et al., 2001a).

As a subject of research and management, water resources are complicated, multiform and sometimes respond to external disturbances in an unpredictable way. Moreover, our measurements are spatially and temporally limited. Data analysis using traditional statistics relies on simplifications, typically in forms of linearization and large sample arguments, both of which can lead to unrealistic estimates of the parameters and especially of their accuracy. On the other hand, traditional physics has encouraged the use of complicated theoretical models which may contain parameters that cannot be estimated accurately with the available data or due to nonlinearities of the model. In the environmental sciences, the results of the analyses are frequently used in decision-making, and the more accurately the uncertainties can be evaluated, the more accurately we can then evaluate the risks of the decisions. Bayesian statistical inference with modern computational methods have provided very useful tools for assessing the uncertainties in environmental studies (Harmon and Challenor, 1996; Kokkonen, 1997; Adams VanHarn, 1998; Omlin and Reichert, 1999; Annan, 2001; Borsuk et al., 2001a).

In this research, we have applied Bayesian data analysis with a computational tool called the Markov chain Monte Carlo (MCMC) method. The usefulness of the method is demonstrated with an example of lake modelling. The time evolution of winter respiration in eutrophic Lake Tuusulanjärvi is estimated, the long-term impact of loading reduction and artificial aeration is assessed and oxygen regime in the lake in a future winter is predicted. Modelling of the interaction between a lake's oxygen regime and all relevant ecological variables may be based on a complicated theoretical framework. The difficulty lies in the design and implementation of experiments that minimize parameter uncertainty, in the quantification of parameter uncertainty, and in the propagation of uncertainty to the predictions. In our particular example, a simplified model was selected to clearly illustrate the multiplicity of uncertainties and the strengths of the MCMC method in tracking them. Limited observational resources, typical in lake management practice, supported the use of a simple model structure. The efficiency and operational status of the artificial oxygenation devices over 30 years could not be traced very accurately and the temperature dependence of respiration was inaccurately known, which caused uncertainties in the inference. Uncertainties in the results were quantified and studied thoroughly and MCMC methods were evaluated with the given example.

We present the basic principles of Bayesian methodology needed for environmental modelling. In addition to the standard MCMC methodology used in some recent papers on environmental problems (Borsuk, 2001; Borsuk et al., 2001a; Qian et al., 2003) we also show useful ideas not commonly implemented. The modelling of the error in the control variables is used to account for all the relevant uncertainties of the phenomenon. We also show how predictive probabilities can easily be calculated for trends in the time evolution of the model. Use of the adaptive MCMC algorithm (Haario et al., 2001) makes it feasible to do calculations with a high dimensional parameter vector, that is, when we have a large (from 50 up) number of unknown parameters.

2. Materials

Lake Tuusulanjärvi is located in the northern hemisphere in southern Finland, latitude 60°26′, longitude 25°03′. The lake is hyper-eutrophic and shallow, its hydrologic and morphometric characteristics are shown in Table 1. The previously mesotrophic state of the lake