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# Rényi dimensions analysis of soil particle-size distributions

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**Abstract**

Dynamism and complexity are terms that have been related to soils. New approaches and points of view have been researched during past decades considering soil's multiple functions and integrating physical, chemical, and biological soil attributes. Also new mathematical models have been explored such as fractal sets and multifractal measures. Characterization of soil particle-size distribution is a key move towards modeling of soils and their behavior with respect to water movement and retention or soil erosion. Modeling soil physical properties related with soil production or degradation and finding indexes of characterization have been set as priorities in many programs related to ecology, sustainable agriculture, etc. Combination of multifractal techniques and laser diffractometry of soil particles have a potential to develop such characterization via analysis of rarely explored regions of the interval of sizes. Multifractal analysis can be applied using Rényi dimensions to detect scaling properties in soil particle distributions. However, specifics of the laser diffraction data require the construction of a new measure with which multifractal techniques may be applied. We developed such a measure and applied the Rényi dimension analysis to 20 soil particle-size distributions obtained by laser diffractometry. These distributions showed, in general, suitable scaling properties, generating Rényi dimensions spectra similar to those obtained for theoretical multifractal properties and showing great variety in their multifractal behavior. Results suggest the applicability of Rényi dimensions spectra to soil characterization for modeling empirical data and generating synthetic data by means of algorithms capable of generating such kind of measures.


*Keywords:* Soil particle-size distributions; Rényi dimensions; Multifractal analysis; Laser diffraction

## 1. Introduction

Concept of soil has been changing from a static point of view towards a more dynamic and complex understanding. Properties, characteristics, and attributes are considered, knowing that describing and managing soils requires a complex approach. As Mermut and Eswaran (2001) pointed out, "in the last few decades, it has become clearer that this task (soil resource management) has to be done in the context of a functional ecosystem." Dynamism and complexity were probably the most repeatedly mentioned terms about soil and quality at the "Defining Soil Quality for a Sustainable Environment" symposium organized by the Soil Science Society of America in 1992. Doran and Parkin (1994) recalled that "assessing soil quality and health is complicated by the need to consider the multiple functions of soil and to integrate the physical, chemical, and biological soil attributes that define soil function (Rodale Inst., 1991; Papendick and Parr, 1992)".

Soil particle distribution is one of the most important physical attributes due to its great influence on soil properties related to water movement, productivity, and erosion (Rieu and Sposito, 1991; Perfect et al., 1996; Giménez et al., 1997). Its characterization and modeling becomes thus an urgent priority in pedology nowadays. The need to find models to represent a soil mass made of solid particles, pores, void spaces, and

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channels in relation to water retention and movement is urgent. On the other hand, particle-size distributions are directly related to soil erosion and degradation. Thus, it is necessary to find a way of characterizing these distributions with parameters capable of detecting the singularity of each distribution.

Many attempts have been made to characterize soil particle distributions. Since 1920, when Baker (Baker, 1920) made one of the earliest attempts to explain the mechanical constitution of arenaceous sediments through the particle diameter as the independent variable, many parametric models have been proposed. Thus, we can find the works of Hatch and Choate (1929), Krumbein (1936, 1938), or Krumbein and Pettijohn (1938) that became basis for studies that were developed between 1940 and 1970. Lognormal distribution (Kolmogorov, 1941; Gardner, 1956) and equations such as Rosin–Rammler relation (Marshall and Quirk, 1950) or Gaudin–Shuhmann power law (Gaudin and Meloy, 1962) were proposed to parameterize particle or fragment distributions.

Hartmann (1969) considered fragmentation as a phenomenon that included terrestrial and interplanetary particles. The following power law:

$$N(m > M) \approx M^{-b} \tag{1}$$

where $N(m > M)$ expressed the accumulated number of fragments with mass over a value $M$, was characterized by exponent $b$. This exponent $b$ took different values depending on the kind of fragmentation that generated the studied particles. Hartmann distinguished between simple fragmentation ($0.5 \leq b \leq 0.8$) due to low expenditure of energy per particle or to inappreciable grinding, and compound fragmentation ($b > 0.8$) implying extreme grinding, crushing, or energy expenditure per particle.

Fractal geometry concepts began to be applied to model soil particle distributions after Mandelbrot (1983) proposed the description and modeling of many natural phenomena using fractals. Few years later, Turcotte (1986) observed that the number of particles of diameter $x$ greater than $X$, $N(x > X)$, and the size of the diameter ($X$) could be fitted with empirical law:

$$N(x > X) \propto X^{-d} \tag{2}$$

where the power law exponent $d$ is a real number $0 < d < 3$.

Power laws expressed in Eqs. (1) and (2) have been a reference for fractal models for the past two decades. There are examples of such models expressing the scale dependence of number of particles with respect to size (Tyler and Wheatcraft, 1989; Perfect and Kay, 1991; Perfect et al., 1992; Rasiah et al., 1992):

$$N(x > X) \propto X^{-D_f} \tag{3}$$

for particles with size $x$ greater than $X$, or the dependence of accumulated distribution of mass for particles under a characteristic size $X$ (Tyler and Wheatcraft, 1992):

$$M(x < X) \propto X^{3-D_f} \tag{4}$$

where $D_f$ is the fractal fragmentation dimension. This fractal exponent has been used to quantify structural changes in soils (Eghball et al., 1993) and has also been the object of many discussions about its utility, meaning, and possible values.

During the last two decades there has also been great interest in developing fractal models for geometric configuration and water movement in soils. Models based on fractal surfaces, with cavities and irregularities retaining water, were presented by de Gennes (1985), Pape et al. (1987), or Toledo et al. (1990). A second class of models was based in the assumption of a fractal pore-size distribution (Tyler and Wheatcraft, 1990; Pachepsky et al., 1995; Perrier et al., 1996) with no supposition about mass geometry and pore-solid interface. In a third kind of models, solid particles and pores followed the same fractal distributions (Rieu and Sposito, 1991; Perfect et al., 1996, 1997; see Giménez et al., 1997 for a review). A new approach to this problem was made by Perrier et al. (1999), Bird et al. (2000), and Perrier and Bird (2002) through the "pore-solid fractal" model. In their works, a soil mass containing solid particles and void spaces was generated, being the solid-pore interface fractal. With respect to primary particles or aggregates distributions, the main issues were the detection of power laws in empirical data, the characterization of the scaling with fractal geometry and dimensions, and the utility of fractal fragmentation dimension to characterize such distributions. Most proposals were based on a single fractal dimension (monofractal) model for soil particle distributions.

Soil is a heterogeneous system of great complexity that has been described as "a more complex medium