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Harvest in a fluctuating environment and conservative harvest for the Fox surplus production model

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Abstract

Relations between optimal yield and abundance in a fluctuating environment and conditions for a conservative level of harvest were obtained for the Fox surplus production model and compared with those for the logistic surplus production model. Environmental variation was included in the optimization of harvest with the Fox surplus production model to obtain a relation in which the maximum sustainable yield (MSY) and biomass at the MSY varied as the environment varied. The relation can be applied for management of fisheries at the optimum levels in a fluctuating environment. For both models there is only one maximum sustainable yield under equilibrium conditions, but in a variable environment the maximum sustainable yield and optimum biomass and effort vary as the environment varies. The results were applied to the blue crab (*Callinectes sapidus*) fishery of the Chesapeake Bay. Although several numerical results for the logistic and Fox models were similar, the parameter estimates were different and the Fox model predicted a much larger decrease in population abundance at the MSY. Harvesting at a conservative level with either the Fox model or the logistic model could increase blue crab abundance substantially with little decrease in harvest. At a conservative level of harvest, there is a 20% increase in biomass with a 6% decrease in yield for the logistic model and a 37% increase in biomass with a 9% decrease in yield for the Fox model. Both the Fox and the logistic surplus production models indicate that the blue crab fishery has been consistently over harvested.

Keywords: Surplus production; Fox model; Gompertz model; Environmental variation; MSY; Blue crab; Chesapeake Bay

1. Introduction

Surplus production models are simple models for harvesting without age structure that assume a population's capacity to increase is some function of population density, and that population density will not change if fish are removed at the same rate as the population's capacity for increase. Early surplus production models were based on the logistic population growth equation and the logistic surplus production model has been widely applied for assessment of fisheries (e.g. Hjort et al., 1933; Graham, 1935; Schaefer, 1954; Pella and Tomlinson, 1969; Quinn and Deriso, 1999). For the logistic surplus production model relations for optimal harvest of populations in a fluctuating environment and for conservative harvests have been developed (Jensen, 2002a,b).

In this study, the relation between maximum sustainable yields and abundance in fluctuating environments and relations for a conservative level of harvest were obtained for the Fox (1970) surplus production model. The Fox (1970) surplus production model is based on the Gompertz equation. The Gompertz equation was originally developed to describe human mortality, but it also can be applied to describe population

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growth, and the Fox (1970) model often fits fisheries data better than the logistic model (e.g. Ricker, 1975; Quinn and Deriso, 1999). In the Fox (1970) model, the maximum rate of population growth occurs at about one-third the carrying capacity rather than at one-half the carrying capacity as in the logistic surplus production model. Density dependence becomes important at a lower population density with the Fox (1970) model than with the logistic model. Both the logistic model and the Fox (1970) model were applied for assessment of the blue crab (*Callinectes sapidus*) fishery of the Chesapeake Bay. Extensive data for the Chesapeake Bay blue crab fishery have been collected and brought together by the Chesapeake Bay Stock Assessment Committee (e.g. Rugolo et al., 2002). The data for application of surplus production models were given to me by Derek Orner, NMFS, NOAA, Chesapeake Bay Office at derek.orner@noaa.gov.

2. Model development

Results for the logistic model that are available in the literature will be briefly summarized for ease of comparison with the Fox (1970) model. Schaefer (1954) obtained the logistic equation for a harvested population as

$$\frac{\mathrm{d}B}{\mathrm{d}t} = \frac{r_{\max}B - r_{\max}B^2}{B_{\inf} - Y}, \tag{1}$$

where, Y is harvest at time t, B is population biomass at time t, $r_{\max}$ is the rate of increase in biomass for a sparse population (per year), and $B_{\inf}$ is the environmental carrying capacity. Eq. (1) assumes that harvest is proportional to abundance and that in the absence of harvesting population growth is logistic. At equilibrium, where $\mathrm{d}B/\mathrm{d}t = 0$, the annual equilibrium harvest Y_e is

$$Y_e = \frac{r_{\max}B_e - r_{\max}B_e^2}{B_{\inf}}, \tag{2}$$

where, B_e is population abundance at equilibrium, $B_e = B_{\inf}(r_{\max} - F)/r_{\max}$, and F is fishing mortality. Population size asymptotically approaches an equilibrium for any value of F such that $0 < F < r_{\max}$. The abundance at which the maximum sustainable yield occurs is $B_{MSY} = B_{\inf}/2$, and the maximum sustainable yield itself, which is the maximum annual yield that the population can sustain in a deterministic environment, is found by substitution of B_{MSY} into the equilibrium harvest equation to give $MSY = r_{\max}B_{\inf}/4$. The instantaneous fishing mortality that results in the MSY is $F_{MSY} = MSY/B_{MSY} = r_{\max}/2$, which is one-half the intrinsic rate of increase. All of the above results are available in the early literature (e.g. Schaefer, 1954; Pella and Tomlinson, 1969; Ricker, 1975).

To develop the Fox surplus production model Fox (1970) assumed that population growth followed the Gompertz equation and this gives the equation for a harvested population as

$$\frac{\mathrm{d}B}{\mathrm{d}t} = k_{\max}B[\ln(B_{\inf}) - \ln(B)] - Y, \tag{3}$$

where the new term $k_{\max}$ is the rate of increase in biomass for a sparse population (per year). Different symbols were used for $k_{\max}$ and $r_{\max}$ because the intrinsic rate of increase $r_{\max}$ is widely associated with the logistic equation. Eq. (3) assumes that harvest is proportional to abundance and that in the absence of harvesting population growth follows the Gompertz equation. At equilibrium, where $\mathrm{d}B/\mathrm{d}t = 0$, the annual equilibrium harvest is (Fox, 1970)

$$Y_e = k_{\max}B_e[\ln(B_{\inf}) - \ln(B_e)]. \tag{4}$$

The abundance at which the maximum sustainable yield occurs is $B_{MSY} = B_{\inf}/e = 0.3679B_{\inf}$ (Fox, 1970). The maximum sustainable yield itself, which is the maximum annual yield that the population can sustain in a deterministic environment, is found by substitution of B_{MSY} into the equilibrium harvest equation to give $MSY = k_{\max}B_{\inf}/e$ (Fox, 1970). The instantaneous fishing mortality that results in the MSY is $F_{MSY} = k_{\max}$ (Fox, 1970). The above results for the Fox (1970) model are available in the literature.

3. Assessment in fluctuating environments

Under equilibrium conditions in a deterministic environment the relation between yield and biomass is a parabola and there is one maximum sustainable yield located at the top of the parabola where $MSY = r_{\max}B_{\inf}/4$ and $F_{MSY} = r_{\max}/2$. In a fluctuating environment, as the environment changes it impacts both the carrying capacity and the rate of increase and there