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# A hierarchy of nonlinear multiparametric models of cloud dynamics and microphysics

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**Abstract**

Three one-dimensional models of pure water (warm) cloud, represented by systems of three, four and five nonlinear ordinary differential equations, respectively, are formulated. They are treated as nonlinear dynamical systems. In all of them, the microphysics is parameterized by Kessler's scheme; the vertical dynamics is modeled by means of the equation of convective motion inside the cloud. The steady solutions for all models are sketched in Appendix A. The evolution of the systems' behaviour of the simplest model is studied. Depending on the parameters' values, the attractor is a fixed point or a limit cycle, or the system is structurally unstable. For some specific combinations of parameters, the system demonstrates chaotic behaviour and the transition to chaos is realized through period-doubling bifurcation. The detailed study of the more complex models, proposed here, is a future task.


*Keywords:* Cloud dynamics and microphysics; Nonlinear dynamical systems; Regular and chaotic regimes.

## 1. Introduction

Clouds in the Earth's atmosphere originate, evolve during hours and disappear. From the synergetic point of view, they represent open nonlinear thermodynamic systems whose evolution is governed by a number of factors acting in one and the same or in opposite

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directions. In general, the cloud is a three-phase system consisting of water vapor, cloud water and ice particles. Particular cases are the warm clouds ($t>0$ °C), also called pure water clouds, as well as the pure ice clouds at very low negative temperatures. The phase transitions of the water in the cloud and the accretion of drops (ice particles) are subject of microphysics; the motions inside the cloud are subject of dynamics. It is obvious that these two parts of the processes cannot be separated, especially when the dynamics is well expressed as it is for the convective type clouds (Cu, Cb), with strong vertical motions inside them. Nevertheless, in theoretical researches, that sort of separation could be used and such "asymptotic" models are not rare. They give possibility to trace out the action of each factor in the model in clear way (form). One could find wide spectrum of models where one of the "asymptotes" prevails—simple one-dimensional (1D) dynamics and more complicated microphysics or contrariwise, as well as intermediate variants, to which case we refer our models.

The main idea comes from Wacker (1992) and has been developed later by Palmer (1995, 1996). The cloud is treated as a nonlinear dynamical system (NDS) and the evolution with time $t$ of its state characteristics $X_i$ is described by a system of first order ordinary differential equations

$$\dot{X}_i = \frac{dX_i}{dt} = f_i(X_1,X_2,\ldots,X_n;v_1,v_2,\ldots,v_m) \tag{1.1}$$

where $v_k$ are control parameters. At least one of the functions $f_i$ ($i=1,\ldots, n$) has to be nonlinear in order to ensure the nonlinearity of the system as a whole.

Eq. (1.1) is built on the base of some physical considerations for the nature of the processes in a cloud. These are balance type equations $\dot{X}=A-B$ (income minus consumption). The microphysics is parameterized using the well known Kessler's scheme (see Rogers, 1976).

## 2. Theoretical base

We suppose to have a pure water (warm) cloud with characteristics $Q_c$ and $Q_p$—mixing ratios of cloud and rain water, respectively. In other words, $Q_c$ and $Q_p$ are representative for the concentrations of the basic (small) cloud droplets and the big drops, which form the rain from the cloud. According to Wacker (1992), the budget equations, valid at each space point in the cloud (or under the cloud when there is precipitation), are in the form:

$$\begin{array}{llll} \dfrac{dQ_c}{dt} = -b_1Q_c & \Big| \; -b_2Q_cQ_p \; \Big| & & \Big| \; +\psi_c \\ \dfrac{dQ_p}{dt} = b_1Q_c & \Big| \; +b_2Q_cQ_p \; \Big| & +P-b_3Q_p & \Big| \; +\Phi_p \\ \qquad\quad \text{I} & \qquad \text{II} & \quad \text{III} & \quad \text{IV} \end{array} \tag{2.1}$$

where $b_i>0$ are constants. The terms in these equations describe different processes as follows:

- I- Autoconversion of the basic cloud droplets (coalescence between them and accretion) by means of turbulent coagulation (effective even in a monodispersed cloud). The electric coagulation could also have contribution;