Available online at www.sciencedirect.com
SCIENCE DIRECT®

Aerosol Science 36 (2005) 1050–1055

Journal of
Aerosol Science

www.elsevier.com/locate/jaerosci

Technical note

An analytical expression for the coagulation coefficient of bipolarly charged particles by an external electric field with the effect of Coulomb force

Wang Lianze, Zhang Xiangrong*, Zhu Keqin

Department of Engineering Mechanics, Tsinghua University, Beijing 100084, China

Received 30 September 2004; received in revised form 13 November 2004; accepted 15 November 2004

Abstract

This article proposes an extension that includes an analytical solution for the coagulation coefficient of bipolarly charged particles by an external electric field with the effect of Coulomb force, based on the known analytical solution for the coagulation coefficient of bipolarly charged particles by Coulomb force without the effect of an external electric field. The proposed solution agreed well with previous numerical results (Journal of Electrostatics 48 (2000) 93).

Keywords: Coagulation coefficient; External electric field; Analytical solution

1. Introduction

The coagulation coefficient of electrically charged particles by Coulomb force without the effect of an external electric field has been derived (Zebel, 1958; Fuchs, 1964). This coagulation coefficient can only be used to describe the Brownian motions between electrically charged (including unipolarly charged and bipolarly charged) particles. Although Koizumi, Kawamura, Tochikubo, and Watanabe (2000) numerically calculated the coagulation coefficient of bipolarly charged particles by an external electric field with the effect of Coulomb force, Koizumi's solution requires complicated computation. Here we unite an

* Corresponding author. Tel.: +86 10 62794155; fax: +86 10 62781824.
E-mail address: zxr01@mails.tsinghua.edu.cn (Z. Xiangrong).

0021-8502/$ - see front matter
doi:10.1016/j.jaerosci.2004.11.008

analytical solution for the coagulation coefficient of bipolarly charged particles by Coulomb force with and without the effect of an external electric field.

2. Theoretical analysis

We assumed that the coagulation coefficient of bipolarly charged particles consists of two parts. One is the coagulation coefficient of bipolarly charged particles by Coulomb force without the effect of an external electric field. The other is that by an external electric field without the effect of Coulomb force. This assumption is based on the fact that Coulomb force strongly works in a short-distance range while the applied external electric field accelerates the coagulation in a long-distance range, thus the two effects work independent of each other.

The coagulation coefficient of bipolarly charged particles by Coulomb force without the effect of an external electric field is given by (Zebel, 1958; Fuchs, 1964)

$$K_{ij} = f_{ij} K_{0ij}, \tag{1}$$

$$K_{0ij} = \frac{2}{3}\frac{\kappa T}{\mu}\left(2 + \frac{d_i}{d_j} + \frac{d_j}{d_i}\right), \tag{2}$$

$$f_{ij} = \frac{\alpha_{ij}}{\exp(\alpha_{ij}) - 1}, \tag{3}$$

$$\alpha_{ij} = \frac{q_i q_j}{2\pi\varepsilon_0 (d_i + d_j)\kappa T}, \tag{4}$$

where K_{0ij} is the Brownian coagulation coefficient of neutral particles (Smoluchowski, 1917), and κ is the Boltzmann constant. T is the absolute temperature, μ the viscosity of the gas, q_i and q_j the charge quantities of the particles with diameters d_i and d_j. Eqs. (1)–(4) are applicable in the continuum regime where the Knudsen number of a particle is small.

If $|\alpha_{ij}| \gg 1$, then $f_{ij} \cong -\alpha_{ij}$. Thus, the coagulation coefficient of bipolarly charged particles by Coulomb force without the effect of an external electric field, Eq. (1), can be approximated by

$$K_{ij} = \frac{q_i q_j}{3\pi\varepsilon_0 (d_i + d_j)\mu}\left(2 + \frac{d_i}{d_j} + \frac{d_j}{d_i}\right). \tag{5}$$

The particle's charge (Koizumi et al., 2000) is given by the sum of the following two charging mechanisms: the field charge, q_{f}, and the diffusion charge, q_{d}:

$$q_{\mathrm{f}} = \frac{3\pi\varepsilon_0\varepsilon_{\mathrm{r}} d^2}{\varepsilon_{\mathrm{r}} + 2} E_{\mathrm{c}} \frac{t/\tau_{\mathrm{f}}}{1 + t/\tau_{\mathrm{f}}}, \tag{6}$$

$$q_{\mathrm{d}} = \frac{2\pi d\varepsilon_0 \kappa T}{e}\ln\left(1 + \frac{t}{\tau_{\mathrm{d}}}\right), \tag{7}$$

where ε_0 and ε_{r} are the permittivity of free space and the relative dielectric constant of the particle. E_{c} is the electric field of the corona discharge, e the unit charge, and τ_{f} and τ_{d} the time constants for field charging and diffusion charging, respectively.