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Population dynamics with a stable efficient equilibrium

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Abstract

We propose a game-theoretic dynamics of a population of replicating individuals. It consists of two parts: the standard replicator one and a migration between two different habitats. We consider symmetric two-player games with two evolutionarily stable strategies: the efficient one in which the population is in a state with a maximal payoff and the risk-dominant one where players are averse to risk. We show that for a large range of parameters of our dynamics, even if the initial conditions in both habitats are in the basin of attraction of the risk-dominant equilibrium (with respect to the standard replication dynamics without migration), in the long run most individuals play the efficient strategy.

Keywords: Evolutionary games; Population dynamics; Evolutionarily stable strategy; Equilibrium selection; Replicator dynamics

1. Introduction

The long-run behavior of interacting individuals can often be described within game-theoretic models. Players with different behaviors (strategies) receive payoffs interpreted as the number of their offspring. Maynard Smith and Price (1973) (see also Maynard Smith, 1982) introduced the fundamental notion of evolutionarily stable strategy. If everybody plays such a strategy, then a small number of mutants playing a different one is eliminated from the population. The dynamical interpretation of an evolutionarily stable strategy was later provided by several authors (Taylor and Jonker, 1978; Hofbauer et al., 1979; Zeeman, 1981). They proposed a system of differential equations, the so-called replicator equations, which describe time-evolution of frequencies of strategies. It is known that any evolutionarily stable strategy is an asymptotically stable stationary point of such dynamics (Weibull, 1995; Hofbauer and Sigmund, 1998, 2003).

We analyse symmetric two-player games with two evolutionarily stable pure strategies and a unique unstable mixed Nash equilibrium. The efficient strategy (called also payoff dominant), when played by the whole population, results in the highest possible payoff (fitness). The risk-dominant one is played by individuals averse to risk. The strategy is risk dominant if it has a higher expected payoff against a player playing both strategies with equal probabilities than the other one (the notion of the risk dominance was introduced and thoroughly studied by Harsányi and Selten (1988)).

Typical example is that of a modified stag-hunt game, where two players choose simultaneously one of two actions: either to join the stag hunt (strategy S) or to go after a hare (strategy H). The strategy S gives the highest possible payoff provided both players join the stag hunt and split the reward. However, when the other player is not loyal and switches to H, then the one playing S receives nothing. The strategy H yields lower but less risky payoffs with a higher payoff when the other player also chooses H. Such a coordination game has two pure Nash equilibria: the efficient one, where all players choose S, and the one in which players are

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averse to risk, and play H. Both strategies are evolutionarily stable and therefore we are faced with the problem of equilibrium selection—which strategy will be played in the long run?

In the replicator dynamics, both strategies are locally asymptotically stable, with the risk-dominant one having a bigger basin of attraction. Here, we propose a dynamical model for which the efficient strategy has a much larger basin of attraction than the risk-dominant one.

We consider a large population of identical individuals who at each time step can belong to one of two different non-overlapping subpopulations or habitats which differ only by their replication rates. In both habitats they play the same two-player symmetric game. Our population dynamics consists of two parts: the standard replicator one and a migration between subpopulations. Individuals are allowed to change their habitats. They move to a habitat in which the average payoff of their strategy is higher; they do not change their strategies.

Migration can help the population to evolve towards an efficient equilibrium. Below we briefly describe the possible mechanism responsible for it. If in a subpopulation, the fraction of individuals playing the efficient strategy A is above its unique mixed Nash equilibrium fraction, then the expected payoff of A is bigger than that of B in this subpopulation, and therefore the subpopulation evolves to the efficient equilibrium by the replicator dynamics without any migration. Let us assume therefore that such fraction is below the Nash equilibrium in both subpopulations. Without loss of generality we assume that initial conditions are such that the fraction of individuals playing A is bigger in the first subpopulation than in the second one. Hence the expected payoff of A is bigger in the first subpopulation than in the second one, and the expected payoff of B is bigger in the second subpopulation than in the first one. This implies that a fraction of A-players in the second population will switch to the first one and at the same time a fraction of B-players from the first population will switch to the second one—migration causes the increase of the fraction of individual of the first population playing A. However, any B-player will have more offspring than any A-player (we are below a mixed Nash equilibrium) and this has the opposite effect on relative number of A-players in the first population than the migration. The asymptotic composition of the whole population depends on the competition between these two processes.

In this note, we derive sufficient conditions for migration and replication rates such that the whole population will be in the long run in a state in which most individuals occupy only one habitat (the first one for the above described initial conditions) and play the efficient strategy.

In Section 2, we introduce the notation and present the class of games we consider. In Section 3, we propose a discrete-time model, obtain its continuous counterpart and prove our results. Section 4 contains a short discussion.

2. Replicator dynamics

We consider two-player symmetric games with two pure strategies and two symmetric Nash equilibria. A payoff matrix is given by

$$U = \begin{array}{ccc} & A & B \\ A & a & b \\ B & c & d, \end{array}$$

where the ij entry, $i,j = A, B$, is the payoff of the first (row) player when he plays the strategy i and the second (column) player plays the strategy j; payoffs of the column player are given by the matrix transposed to U.

An assignment of strategies to both players is a Nash equilibrium, if for each player, for a fixed strategy of his opponent, changing the current strategy cannot increase his payoff.

In order to use our payoff matrix in the dynamics of reproducing species, we assume that all payoffs are nonnegative. We also assume that $a>d>c$, $d>b$, and $a+b<c+d$ which implies in particular that (A,A) and (B,B) are two strict Nash equilibria hence both A and B are evolutionarily stable. The last inequality means that an expected payoff of B against a player playing both strategies with equal probabilities is higher than that of A. We address the problem of equilibrium selection between the payoff-dominant (efficient) equilibrium (A,A) and the risk dominant (B,B). Our assumptions imply also that a B-player receives a maximal payoff when he plays against another B-player.

As an example, consider the following payoffs: $a = 4$, $b = 0$, $c = 2$, $d = 3$. The strategy profile (A,A) is more risky than (B,B) since from the point of view of the row player, a deviation by the column player in (A,A) results in a payoff loss of 4 units versus a loss of 1 in (B,B).

In the replicator dynamics, individuals of a large population are matched many times to play the above described stage game. Let x be the fraction of the population playing A. The expected payoff of A is given by $f_A = ax + b(1-x)$ and that of B by $f_B = cx + d(1-x)$. A mixed Nash equilibrium strategy is such x^* that the above two expected values are equal, hence $x^* = (d-b)/(d-b+a-c)$. In the standard replicator dynamics, the rate of change of x is proportional to the expected payoff of A. The corresponding differential equation reads (Weibull, 1995;