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Journal of Theoretical Biology 232 (2005) 47–53
Journal of Theoretical Biology
www.elsevier.com/locate/yjtbi

Equilibrium selection in evolutionary games with random matching of players

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Received 3 March 2004; received in revised form 22 July 2004; accepted 22 July 2004
Available online 8 September 2004

Abstract

We discuss stochastic dynamics of populations of individuals playing games. Our models possess two evolutionarily stable strategies: an efficient one, where a population is in a state with the maximal payoff (fitness) and a risk-dominant one, where players are averse to risks. We assume that individuals play with randomly chosen opponents (they do not play against average strategies as in the standard replicator dynamics). We show that the long-run behavior of a population depends on its size and the mutation level.

Keywords: Population dynamics; Evolutionarily stable strategy; Equilibrium selection; Stochastic stability

1. Introduction

The long-run behavior of interacting individuals can often be described within game-theoretic models. The basic notion here is that of a Nash equilibrium. This is a state of population—an assignment of strategies to players—such that no player, for fixed strategies of his opponents, has an incentive to deviate from his current strategy; the change can only diminish his payoff. Nash equilibrium is supposed to be a result of decisions of rational players. Maynard Smith (1974, 1982) has refined this concept of equilibrium to include the stability of Nash equilibria against mutants. He introduced the fundamental notion of an evolutionarily stable strategy. If everybody plays such a strategy, then the small number of mutants playing a different strategy is eliminated from the population. The dynamical interpretation of the evolutionarily stable strategy was later provided by several authors (Taylor and Jonker, 1978; Hofbauer et al., 1979; Zeeman, 1981). They proposed a system of differential or difference equations, the so-called replicator equations, which describe the time evolution of frequencies of strategies. It is known that any evolutionarily stable strategy is an asymptotically stable stationary point of such dynamics (Hofbauer and Sigmund, 1988; Weibull, 1997).

Here we will discuss a stochastic adaptation dynamics of a population of players interacting in discrete moments of time. We will analyse two-player games with two strategies and two evolutionarily stable strategies. The efficient strategy (also called payoff dominant) when played by the whole population results in its highest possible payoff (fitness). The risk-dominant one is played by individuals averse to risks. The strategy is risk dominant if it has a higher expected payoff against a player playing both strategies with equal probabilities. We will address the problem of equilibrium selection–a strategy which will be played in the long run with a high frequency.

We will review two models of adaptive dynamics of a population of a fixed number of individuals. In both of them, the selection part of the dynamics ensures that if the mean payoff of a given strategy at the time t is bigger than the mean payoff of the other one, then the number

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0022-5193/$ - see front matter
doi:10.1016/j.jtbi.2004.07.019

of individuals playing the given strategy should increase in $t+1$. In the first model, introduced by Kandori et al. (1993), one assumes (as in the standard replicator dynamics) that individuals receive average payoffs with respect to all possible opponents—they play against the average strategy. In the second model, introduced by Robson and Vega-Redondo (1996), at any moment of time, individuals play only one game with randomly chosen opponents. In both models, players may mutate with a small probability; hence the population may move against a selection pressure. To describe the long-run behavior of such stochastic dynamics, Foster and Young (1990) introduced a concept of stochastic stability. A configuration of a system is stochastically stable if it has a positive probability in the stationary state of the above dynamics in the limit of no mutations. It means that in the long run we observe it with a positive frequency. Kandori et al. (1993) showed that in their model, the risk-dominant strategy is stochastically stable—if the mutation level is small enough we observe it in the long run with the frequency close to one. In the model of Robson and Vega-Redondo (1996), the efficient strategy is stochastically stable. It is one of the very few models in which an efficient strategy is stochastically stable in the presence of a risk-dominant one. The population evolves in the long run to a state with the maximal fitness.

The main goal of our paper is to investigate the effect of the number of players on the long-run behavior of the Robson–Vega-Redondo model. We will discuss parallel and sequential dynamics, and the one, where each individual enjoys each period a revision opportunity with some independent probability. We will show that in the last two dynamics, for any arbitrarily low but a fixed level of mutations, if the number of players is sufficiently big, a risk-dominant strategy is played in the long run with a frequency close to one—a stochastically stable efficient strategy is observed with a very low frequency. It means that when the number of players increases, the population undergoes a transition between an efficient payoff-dominant equilibrium and a risk-dominant one. We will also show that for some range of payoff parameters, stochastic stability itself depends on the number of players. If the number of players is below a certain value (which may be arbitrarily large), then a risk-dominant strategy is stochastically stable. An efficient strategy becomes stochastically stable only if n is large enough, as proved by Robson and Vega-Redondo (1996).

In Section 2, we introduce Kandori–Mailath–Rob and Robson–Vega-Redondo models and review their main properties. In Section 3, we analyse the Robson–Vega-Redondo model in the limit of the infinite number of players and show our main results. Discussion follows in Section 4.

2. Models of adaptive dynamics with mutations

We will consider a finite population of n individuals who have at their disposal one of the two strategies: A and B. At every discrete moment of time, $t = 1, 2, \ldots$, they are randomly paired (we assume that n is even) to play a two-player symmetric game with payoffs given by the following matrix:

$$U = \begin{array}{ccc} & A & B \\ A & a & b \\ B & c & d \end{array}$$

where the ij entry, $i,j = A, B$, is the payoff of the first (row) player when he plays the strategy i and the second (column) player plays the strategy j. We assume that both players are the same and hence payoffs of the column player are given by the matrix transposed to U; such games are called symmetric.

We assume that $a > c$ and $d > b$, therefore, both A and B are evolutionarily stable strategies, and $a + b < c + d$, so the strategy B has a higher expected payoff against a player playing both strategies with the probability $\frac{1}{2}$. We say that B risk dominates the strategy A (the notion of the risk-dominance was introduced and thoroughly studied by Harsányi and Selten (1988)). We also assume that $a > d$; hence we have a selection problem of choosing between the risk-dominant B and the so-called payoff-dominant or efficient strategy A.

At every discrete moment of time t, the state of our population is described by the number of individuals, z_t, playing A. Formally, by the state space we mean the set

$$\Omega = \{z, 0 \leqslant z \leqslant n\}.$$

Now we will describe the dynamics of our system. It consists of two components: selection and mutation. The selection mechanism ensures that if the mean payoff of a given strategy, $\pi_i(z_t)$, $i = A, B$, at the time t is bigger than the mean payoff of the other one, then the number of individuals playing the given strategy should increase in $t+1$. In their paper, Kandori et al. (1993) write

$$\pi_A(z_t) = \frac{a(z_t - 1) + b(n - z_t)}{n - 1},$$

$$\pi_B(z_t) = \frac{cz_t + d(n - z_t - 1)}{n - 1}, \tag{2.1}$$

provided $0 < z_t < n$.

It means that in every time step, players are paired infinitely many times to play the game or equivalently, each player plays with every other player and his payoff is the sum of corresponding payoffs. This model may be therefore considered as an analog of replicator dynamics for populations with fixed numbers of players.