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General methodology for nonlinear modeling of neural systems with Poisson point-process inputs

V.Z. Marmarelis *, T.W. Berger

University of Southern California, Biomedical Engineering, 3650 McClintock Avenue, Olin Hall 500, Los Angeles, CA 90089-1415, United States

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Abstract

This paper presents a general methodological framework for the practical modeling of neural systems with point-process inputs (sequences of action potentials or, more broadly, identical events) based on the Volterra and Wiener theories of functional expansions and system identification. The paper clarifies the distinctions between Volterra and Wiener kernels obtained from Poisson point-process inputs. It shows that only the Wiener kernels can be estimated via cross-correlation, but must be defined as zero along the diagonals. The Volterra kernels can be estimated far more accurately (and from shorter data-records) by use of the Laguerre expansion technique adapted to point-process inputs, and they are independent of the mean rate of stimulation (unlike their P–W counterparts that depend on it). The Volterra kernels can also be estimated for broadband point-process inputs that are not Poisson. Useful applications of this modeling approach include cases where we seek to determine (model) the transfer characteristics between one neuronal axon (a point-process 'input') and another axon (a point-process 'output') or some other measure of neuronal activity (a continuous 'output', such as population activity) with which a causal link exists.

Keywords: Volterra kernels; Wiener kernels; Nonlinear modeling; Poisson inputs; Point-process inputs; Neural systems; Neuronal modeling

* Corresponding author. Tel.: +1 213 740 0841; fax: +1 213 740 0343.
E-mail address: marmarelis@hotmail.com (V.Z. Marmarelis).

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1. Introduction

Modeling studies of neural systems that are stimulated by temporal sequences of action potentials (spike trains) can be placed in the general mathematical framework of functional expansions, due to the intrinsic system nonlinearities [1–3,7–9,12,10,13,11,14,17–19]. When these action potentials are idealized as impulses of fixed intensity (Dirac delta functions in continuous time or Kronecker deltas in discrete time), they can be represented mathematically in a stochastic context by 'point processes': a specific class of random processes that are formed by sequences of identical impulses representing random, instantaneous, and identical events.

These modeling studies must be properly placed in a stochastic context since actual experiments of neural systems are burdened inevitably by systemic or measurement noise and other uncontrollable factors introducing variations in the experimental data that are viewed as stochastic processes. Furthermore, the digital processing of the experimental data necessitates analysis in a discrete-time framework, although the actual biological processes take place in continuous time. Proper discretization of a sequence of action potentials requires that the sampling interval (bin-width) be approximately equal to the refractory period in order to allow the representation of each continuous-time action potential by one and only one Kronecker delta in the respective bin (the intensity of the latter ought to be the integrated area under the action potential divided by the bin-width T).

Thus, in practice, each sequence of action potentials is represented by a discrete-time point-process

$$\chi(n) = A\sum_{i=1}^{K}\delta(n - n_i), \tag{1}$$

where n denotes the discrete-time index ($t = nT$), A is the intensity of the Kronecker delta and n_i is the time-index of the ith event (i.e., discretized timing of the ith action potential). Note that this process has K events over the available data record of N bins, i.e., the mean rate of this point-process is (K/N). We seek to address the problem of neural system modeling from input–output data, where the system output $y(n)$ may be continuous or a point-process but the system input $\chi(n)$ is always a point-process described as in Eq. (1).

The potential utility of this modeling approach to the study of actual neural systems is found in those cases where we seek to determine (model) the transfer characteristics between one neuronal axon (a point-process 'input') and another axon (a point-process 'output') or some other measure of neuronal activity (a continuous 'output', such as population activity) with which a causal link exists (or must be confirmed). Another set of potential applications with point-process inputs are neural prostheses and in vitro studies (like the original studies in the hippocampal slices by Berger, Sclabassi and their associates that provided the initial motivation for this type of modeling work).

2. The modeling problem

In the general formulation of the modeling problem, we seek an explicit mathematical description of the causal functional F that maps the input past (and present) upon the present value of the output,

$$y(n) = F[\chi(n'), n' \leqslant n]. \tag{2}$$