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Threshold policies control for predator–prey systems using a control Liapunov function approach ☆

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Abstract

The stability of predator–prey models, in the context of exploitation of renewable resources, subject to threshold policies (TP) is studied in this paper using the idea of backstepping and control Liapunov functions (CLF) well known in control theory, as well as the concept of virtual equilibria. TPs are defined and analysed for different types of one and two species predator–prey models. The models studied are the single species Noy-Meir herbivore-vegetation model, in a grazing management context, as well as the Rosenzweig–MacArthur two species predator–prey model, in a fishery management context. TPs are shown to be versatile and useful in managing renewable resources, being simple to design and implement, and also yielding advantages in situations of overexploitation.

Keywords: Predator–prey systems; Control Liapunov function; Sustainable yield; Variable structure system; Threshold policy; Global stability; Virtual equilibrium

1. Introduction

Grazing management refers to the manipulation of livestock to systematically control periods of grazing and no grazing (usually termed *deferment* or *rest*). The primary objectives are to control the effects of grazing at the individual plant level in order to protect soil watershed and improve livestock production (Heitschmidt and Stuth, 1991). In grazing management, it is possible to control the consumption of the herbivore (predator) by allowing or not allowing grazing. A mathematical model that is much used in the study of herbivore grazing was proposed by Noy-Meir (1975) and will be examined in this paper. The Noy-Meir model describes vegetation growth under the assumption that it is subject to the action of a constant herbivore population. In common with most other single species models in the literature, it has a logistic growth term, and a consumption term that models the action of the herbivore.

In the grazing management context, when a scheme such as *short duration* or *deferred rotation* is used, it means that the consumption term is being switched on (when grazing of a particular paddock is allowed) and off (when the livestock is fenced out of the paddock) (Heitschmidt and Stuth, 1991). Another possibility arises in grazing models of coral reefs which can flip between coral- and algae-dominated states. It has been postulated that the interplay between herbivorous fish and algae is an important factor in determining the flipping dynamics, since removal of the fish might induce

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an algae-dominated state. Such a proposal, based on the Noy–Meir model, of herbivore fish–algae dynamics, has been made in Crépin (2002).

Similarly, *a fishing policy* refers to the management of fish populations by systematically controlling the period, intensity and type of fishing. Once again, the primary objectives are to maximize productivity, without depleting or driving the stocks to extinction. In a fishery model, where two species are modelled and fishery of the prey species is of interest, it is generally not possible to control the consumption of the prey by the predator species, referred to as *endogenous consumption*, but control may be exercised by the removal (fishing) of a certain quantity of the prey species, which we will refer to as *exogenous consumption*. A mathematical model that is much used in the study of fishery is called the Gordon–Schaefer model (Clark, 1976, 1985; Imeson et al., 2002) and its variants (Collie and Spencer, 1993) are also studied in this paper. The Gordon–Schaefer model proposes a logistic growth term for the fish. Crépin (2002) proposes, in addition, an endogenous predation term (corresponding to herbivorous fish prey eaten by fish predators, whose population is assumed to be constant in time, thus entering as a predation rate), and a removal rate (corresponding to the removal of herbivorous fish by an exogenous agent (man)). Thus it is often true that is possible to introduce an exogenous control into either the prey or predator dynamics.

Given the complexity of ecosystem dynamics, it is only feasible to use very simple control actions. A commonly used and implementable control is to allow removal of the predator, when its density exceeds a specified threshold level—a good example of this is in a harvesting or fishing context (Collie and Spencer, 1993; Quinn and Deriso, 2000).

This paper is concerned with the introduction of such an exogenous control into one- and two-dimensional population dynamical system models. The overall objective is to develop a systematic way of designing simple implementable controls that drive the dynamical systems to a desired globally stable equilibrium, in which a desired population level is maintained and, in the case of two population models, coexistence of predator and prey population models should result, i.e. the proposed control must avoid the extinction of the species, even under certain conditions of overexploitation of the species.

This objective is attained by using the control Liapunov function (CLF) approach from the control literature (Sontag, 1989) in order to choose the control. The objective of keeping the control as simple as possible so as to be implementable is achieved by using on–off controls that are activated when a certain threshold population density increases beyond a given level. This threshold population density may be a population density itself or derived in some simple manner from these densities. The choice and positioning of the threshold is guided by the CLF as well as the concept of real and virtual equilibria introduced in Costa et al. (2000). Finally, in the case of two population predator–prey models, the simplicity of the control is achieved by introducing the control into only one of the species dynamics, and, in this case, inspired by the method of backstepping (Sepulchre et al., 1997), a CLF is used to design the control. This combination of concepts—real and virtual equilibria, CLFs, on–off control and backstepping—to introduce a globally stable equilibrium into a nonlinear dynamical population model is novel in this context and is one of the contributions of the paper. Finally, it is shown that the type of control considered in this paper has advantages in situations where overexploitation of the populations occurs, which is important in the resource management context.

1.1. Previous work and preliminaries

In the context of fishing management, Collie and Spencer (1993) introduced a so-called *threshold policy* (TP), which is intermediate between the well-known constant escapement and constant harvest rate policies (Quinn and Deriso, 2000). A TP is defined as follows: if abundance is below the threshold level, there is no harvest; above the threshold, a constant harvest rate is applied. The TP is also referred to as an on–off control and is a special and simple case of variable structure control in the control literature (Utkin, 1978, 1992; Filippov, 1988; Edwards and Spurgeon, 1998).

We establish a standard notation for a TP (see Fig. 1), denoting it as the function $\phi(\tau)$ defined as follows:

$$\phi(\tau) = \begin{cases} 1 & \text{if } \tau > 0, \\ 0 & \text{if } \tau < 0, \end{cases} \tag{1}$$

where τ is the threshold that should be chosen adequately, depending on the problem to be solved. The case of τ exactly equal to zero, for which the value of ϕ is not defined in (1) is discussed further below in Definition 2.

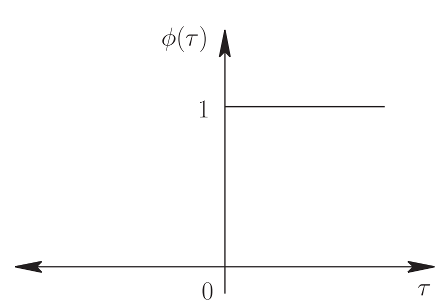

Fig. 1. Threshold policy.