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Modelling thin film flow with erosion and deposition

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Abstract

A model is developed for the laminar flow of a thin film mixture made of water and particles over a surface with roughness of the same order as the film height. Particle erosion and deposition are also considered and modelled with two separate equations which are coupled to the fluid flow. This leads to a self-regulated erosion/deposition model, not requiring an explicit value for the transport capacity of the flow, this parameter appears analytically in the model. This approach is tested against a standard model for the erosion/deposition process. Algorithms are also developed to evaluate the required non-standard roughness parameters from experimental measurements and a numerical test of the model is carried out.

Keywords: Laminar flow; Rough surface; Thin film; Erosion; Deposition; Transport capacity

1. Introduction

In order to understand and therefore control, or at least predict overland flow, it is crucial to develop an adequate fluid mechanical model of the process. Such a model must describe the fluid flow over a rough surface subject to erosion, particle transport and deposition, rainfall and infiltration into the soil. To further complicate matters, the presence of particles in the flow will affect the fluid properties which in turn will influence the erosion and deposition rates. The present work, entirely based on the fluid flow study, offers a solution that takes these various aspects into account.

When modelling overland flows, the majority of authors have investigated the turbulent flow regime both theoretically and experimentally, see [2,16,17] for example. Recently attention has been focused on the laminar flow regime. Myers [21,22] studies the flow of clean water over a rough surface. This approach will be here generalised for agricultural overland flow. Rain, infiltration and soil particle movement will be included in the model. Rain and infiltration will be incorporated through appropriate boundary conditions. Particles will be modelled by considering the mixture of water and particles as a single-phase fluid. The properties of such a mixture have already been studied [5,10] and the equation governing the fluid flow on a rough surface may be modified accordingly.

Soil particles may be eroded by either rain-splash or the shear forces created by the flow [13]. In the present work, the erosion by shear forces only will be considered, rainfall detachment will not be studied. Once detached from the ground, particles are transported by the water and fall back progressively towards the soil surface under the action of gravity. The presence of particles in the fluid mixture affects the erosion and deposition processes. This effect is traditionally incorporated in the model via a parameter known as the transport

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Nomenclature

c	particle volumetric concentration
d	particle diameter
f_1, f_2	gravity and dynamic viscosity correction coefficients
g	gravity
h	fluid height
k	maximum roughness height
k_c	flow entrainment coefficient
n	Manning number
$\mathbf{n}$	normal vector
p	pressure
r	rainfall rate
s	surface height
t	time
$\mathbf{u} = (u, v, w)$	velocity components
u_s	particle settling velocity
(x, y, z)	Cartesian coordinates
$(\mathbf{x}, \mathbf{y}, \mathbf{z})$	unit vectors in the Cartesian directions
A, B	velocity constants
C^*	transport capacity
D	deposition rate
E	erosion rate
F	fraction of stream power used in erosion
G	Bond number
H	height scale
I	infiltration rate
L	length scale
P	pressure scale
Q	fluid flux
Re	Reynolds number
T	time scale
U	velocity scale
U^*	Manning velocity
$\mathbf{V}$	rain droplets velocity
$\mathscr{V}$	rain droplets velocity scale

Greek symbols

α	soil angle to the horizontal
γ	soil composition coefficient
ϵ	aspect ratio of the flow
ζ	correction factor for the settling velocity
μ	dynamic viscosity
ρ	density
ρ_a	density of water in the air
σ	density of wet sediment
τ	shear stress
τ_c	critical shear stress
ϕ	soil porosity
ω	roughness coefficient
Λ	gravity influence on the velocities

Subscripts

e	experimental
i	particle class
s	soil
w	water
x	x-direction
y	y-direction
z	z-direction

capacity [8]. This quantity is notoriously difficult to evaluate and the present work offers an alternative to this approach by considering the water and particle mixture as a single-phase fluid. The transport capacity then becomes redundant: the consequences of the presence of particles in the flow are automatically integrated in the model, that is then significantly simplified. This method was first introduced by Hairsine and Rose [13] and the model was regularly further developed, see for example [12,26,28,29]. A similar approach is followed in the present work.

In the following sections, the flow model is developed first. As is standard for thin film laminar flow, the problem reduces to solving a single equation for the film height [20]. Expressions for the flux, velocity and shear stress are obtained in terms of the film height and roughness characteristics of the soil. These analytical formulae are then used to model the erosion and deposition processes.

To validate the approach, the analytical value obtained for the transport capacity will be compared with results calculated in the well established model by Hairsine and Rose [13] for the erosion/deposition process. The fluid model uses a non-standard roughness coefficient. This parameter was first introduced by Myers [21,22]. It describes the effect on the flow of the roughness of the soil surface. An algorithm is presented to evaluate this key parameter for the model. This will be tested on experimental data measured by Phelps [27].

To conclude the study, a simple example on a flat inclined surface will be fully detailed. The governing equations are discretised and the outcome of numerical simulations will be discussed.

2. Flow model

The equation governing the fluid flow is now derived. The fluid is a mixture of water and soil particles. Each class of particle is represented by its volumetric concentration in the fluid, c_i. The total particle volumetric concentration is then $c = \sum c_i$. It is assumed that the