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Internal pressure errors in sigma-coordinate ocean models—sensitivity of the growth of the flow to the time stepping method and possible non-hydrostatic effects

Jarle Berntsen[a,b,*], Gunnar Furnes[b,c]

[a]Department of Mathematics, University of Bergen, Johs. Bruns gt 12, N-5008 Bergen, Norway
[b]The Bergen Center for Computational Science, UNIFOB, Museplass 2, N-5020 Bergen, Norway
[c]Norsk Hydro, E & P Research Centre, P.O. Box 7190, N-5020 Bergen, Norway

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Abstract

Sigma coordinate ocean models, or models based on more generalised topography following coordinate systems, are presently widely used in oceanographic studies. Terrain following models are attractive because of their abilities to resolve the surface and bottom layers. However, the internal pressure gradient estimation is problematic in such models, and artificial pressure gradients may create artificial flow.

In fine resolution studies where topography and stratification is well resolved, the artificial flow due to erroneous pressure gradients is small because the errors typically converge to zero with the square of the grid size. In coarse resolution studies, however, there may still be large erroneous pressure gradients that may create strong artificial flow. To avoid this, larger horizontal viscosities than one would like to apply from physical considerations, are applied in most studies so far. This means that smaller scale phenomena in the flow will be poorly represented.

The response of the flow to the pressure errors in long time simulations with low horizontal viscosity is very important. In this study it is demonstrated that the choice of the time stepping method may affect the growth of the erroneous flow considerably. Increasing the implicit nature of the time stepping method, will generally reduce the growth.

Recently many oceanic model studies have been performed with models that allow non-hydrostatic pressure effects. In the present study it is shown that when allowing the flow to adjust to non-hydrostatic pressure, the growth of the flow due to internal pressure errors may be strongly affected.

Keywords: Numerical ocean modelling; Sigma coordinates; Internal pressure; Time-stepping; Non-hydrostatic

*Corresponding author. Department of Mathematics, University of Bergen, Johs. Bruns gt 12, N-5008 Bergen, Norway. Tel.: 47 55 584854; fax: 47 55 589672.

E-mail addresses: jarle.berntsen@mi.uib.no, jarleb@mi.uib.no (J. Berntsen), gunnar.furnes@hydro.com (G. Furnes).

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1. Introduction

Sigma coordinate ocean models, or models based on more generalised topography following coordinate systems, are presently widely used in oceanographic studies. From the home page for the Princeton Ocean Model (POM) (Blumberg and Mellor, 1987) we for instance find that this model alone has more than 1100 users from 59 countries in year 2002. The abilities of such models to resolve the bottom and surface layers are attractive. However, the controversy over internal pressure errors in sigma coordinate ocean models is still worrying to some of the users. Geophysical flow is to a large extent determined by the balance between internal pressure and the Coriolis force. If there are large errors in the estimates of internal pressure, the estimated circulation will be wrong.

To our knowledge Haney, 1991 was the first to focus on the problem with the pressure gradient force over steep topography in sigma coordinate ocean models and hydrostatic consistency. The literature in numerical oceanography traces back to earlier publications in numerical meteorology on pressure gradients in topography following coordinates, and Haney also summarises some of this literature. See for instance Sundquist (1975, 1976), Janjic (1977), Mesinger (1982) and Mesinger and Janjic (1985).

The source of the problem is that in σ-coordinates, the x-component of the internal pressure is written

$$\left.\frac{\partial\rho}{\partial x}\right|_z = \frac{\partial\rho}{\partial x} - \frac{\sigma}{H}\frac{\partial H}{\partial x}\frac{\partial\rho}{\partial\sigma}, \tag{1}$$

where x is the horizontal coordinate, z the vertical coordinate, ρ the density, H the depth, and $\sigma \equiv z/H$. Near steep topography the two terms on the right may be large, comparable in magnitude, and often opposite in sign. This may cause large errors in the estimates of the internal pressure.

Introducing the buoyancy $b = \rho g/\rho_0$ where g is the gravity constant and ρ_0 a constant reference density Mellor et al. (1994) using Taylor expansions studied the discretisation error for the 2nd order internal pressure method used in POM. The error is to leading order

$$\mathrm{E}\left(\frac{\delta_x b}{\delta x}\right) = \frac{H}{4}\frac{\delta_x H}{\delta x}\left(\frac{\partial^2 b}{\partial z^2}\right)\left\{(\delta\sigma)^2 - \sigma^2\left(\frac{\delta_x H}{H}\right)^2\right\}. \tag{2}$$

The error term (2) shows that the error will decrease as $\delta\sigma$ and $\delta_x H$ both tend to zero. It also shows that for a given σ and $\delta_x H/H$ the error will be limited as $\delta\sigma$ goes to zero. On the other hand to add more layers does not improve the quality when the $\delta_x H/H$ term already is the limiting factor. The error term (2) give hope in the sense that as computers get more powerful and we may reduce $\delta\sigma$ and $\delta_x H/H$, we may sooner or later get to the point where we do not have to worry about internal pressure errors. However, for the foreseeable future the horizontal resolution in models covering large geographical areas and containing steep shelf slopes and underwater seamounts will not be sufficient and the internal pressure errors may still be worrying. For cases where the buoyancy is a function of z only, and for idealised density profiles, the internal pressure errors may be translated to erroneous geostrophic velocities. This is done for instance in Haney (1991) and in Slørdal (1995). It is not hard to demonstrate errors greater than $0.10\,\mathrm{m\,s^{-1}}$ in geostrophic velocities.

Mellor et al. (1994) showed that in diagnostic experiments, the pressure errors are maintained in time as an input of potential energy and the mean kinetic energy does not tend to zero even if the system is not externally forced. In prognostic experiments, the density field is allowed to be advectively adjusted, and it is demonstrated that for this case the numerical potential energy may go to zero and that the mean kinetic energy accordingly will die out.

Beckmann and Haidvogel (1993) studied flow trapped to a seamount. For the non-forced case and horizontal stratification, eight eddies around the seamount were growing for larger Burger numbers and they did not die out prognostically. Thus for 3D cases allowing vorticity to develop, it was demonstrated that the effects of initial internal pressure errors may even grow in time.