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Application of a coupled discontinuous–continuous Galerkin finite element shallow water model to coastal ocean dynamics

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Abstract

A coupled discontinuous–continuous Galerkin (DG–CG) shallow water model is compared to a continuous Galerkin generalized wave-continuity equation (GWCE) based model for the coastal ocean, whereby local mass imbalance typical of GWCE-based solutions is eliminated using the coupled DG–CG approach. Two mass imbalance indicators for the GWCE-based model are presented and analyzed. The indicators motivate discussion on the suitability of using a GWCE-based model versus the locally conservative coupled DG–CG model. Both realistic and idealized test problems for tide, wind, and wave-driven circulation form the basis of the study. For the problems studied, coupled DG–CG solutions retain the robustness of well-documented solutions from GWCE-based models and also capture the dynamics driven by small-scale, highly advective processes which are problematic for GWCE-based models. Issues associated with the coupled DG–CG model are explored, including increased cost due to increased degrees of freedom, the necessary application of slope limiters, as well as the actual coupling process.

Keywords: Shallow water equations; Discontinuous Galerkin methods; Coupled finite element methods; Generalized wave continuity equation; ADCIRC; Coastal ocean models; Mass error; Tidal dynamics; Rip currents; Bahamas, Bight of Abaco

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1. Introduction

Since the construct of the wave-continuity equation (Lynch and Gray, 1979) two decades ago, finite element-based shallow water equation models have generated robust computations of tide and surge dynamics. These models typically utilize unstructured triangular grids which permit a large degree of flexibility in representing the complexities of the coastal ocean associated with convoluted shorelines and steep gradients in either currents or bathymetry and additionally accommodate the need for remote oceanic forcing. The recent availability of massively parallel computing resources has pushed the application of these models to higher and higher resolutions (down to meters) while capturing a more diverse array of smaller-scale physical processes present in the coastal ocean. One manifestation of these high resolution applications has been an increased occurrence of erroneous circulation fields whose errors can be attributed to the lack of enforcement in local mass conservation.

Mass imbalances in the transient solutions obtained from wave-continuity based finite element shallow water equation models were observed independently in 1992 by Kolar and Westerink (2000). A common source for these errors was determined to be the non-linear terms, particularly the advection terms. Since that time a number of efforts have been directed at removing or minimizing the presence of mass error in finite element-based computations. Improvements in the conservative properties of these models have come by applying a consistent treatment of the advective terms between the continuity and momentum equations as suggested by Kolar et al. (1994). A further reduction in mass error was realized by using mass conserving forms of the flux boundary conditions as proposed by both Lynch (1985) and Kolar et al. (1994). An entirely separate approach sought to mitigate mass conservation issues by constructing an "optimal" triangular mesh over which mass error is minimally uniform (Zhang and Baptista, 2000).

To date, mass errors still plague the solutions for some applications of finite element-based shallow water models that utilize the generalized wave continuity equation (GWCE) in place of the primitive form of the continuity equation. The GWCE (Kinnmark, 1986), a generalization of the wave-continuity equation formulation introduced by Lynch and Gray (1979), eliminates spurious oscillations that appeared in early finite element models that were based on the primitive continuity equation and used the same linear bases for both elevation and velocity. The absence of spurious spatial oscillations in the GWCE formulation is attributed to the monotonic dispersion relation that prevents the spurious, zero frequency, oscillations from forming while short wavelength modes ready propagate through the domain and do not accumulate energy (Walters, 1983). One consequence of using the GWCE has been the removal of the explicit enforcement of local (element by element) mass conservation.

While continuous Galerkin (CG) finite element-based shallow water equation models formally require global conservation (Lynch, 1985; Lynch and Holboke, 1997), the lack of local mass conservation remains problematic particularly for simulating small scale, highly advective flows and/or transport dynamics. One example is provided by Oliveira et al. (2000) who report the impact of flow mass errors on the fidelity of the computed mass transport for a tidally-driven application. For three-dimensional circulation problems, Luettich et al. (2002) demonstrate that significant errors in the vertical velocity result when mass errors are present in horizontal current fields. Thus, GWCE-based models have difficulty accurately capturing the vertical mixing and transport processes necessary to simulate flows dominated by density-driven dynamics. Of course, freshwater