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A comparison of weakly and fully non-linear models of the shoaling of a solitary internal wave

Richard Justin Small [a,*], Robert Philip Hornby [b]

[a] *International Pacific Research Center, School of Earth and Ocean Sciences and Technology, University of Hawaii, 2525 Correa Rd., Honolulu, Hawaii 96822, USA*
[b] *Defence Science and Technology Laboratory, Building A33, Winfrith Technology Centre, Dorchester DT2 8WX, UK*

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Abstract

This study investigates the behaviour of internal solitary waves crossing a continental slope in the presence of a seasonal thermocline. Comparisons are made between a fully non-linear computational fluid dynamics (CFD) model, and weakly non-linear theory. Previous observations suggested that the amplitudes of solitary waves are capped as they pass across the continental slope, which may be due to laminar dynamics, or due to the effect of turbulence. Across the continental slope, CFD and second order variable depth KdV (vEKdV) predictions agree well with observations of a limited change in solitary wave amplitude. First order variable depth KdV theory overpredicts the final amplitude significantly. In terms of the wave shape, the CFD modeled wave changes from a KdV shape in deep water towards an EkdV solution in shallow water, as observations suggest. The phase speed of the CFD and vEKdV waves are similar to that observed in waters of 400–500 m deep, but are slightly lower than observed in 140 m depth. CFD predictions using a standard k, ε turbulence model showed that turbulence had little effect on the amplitude. These preliminary results indicate that in this situation wave capping is due to laminar, large amplitude solitary wave dynamics and is independent of turbulent mixing.

1. Introduction

Internal solitary waves (ISWs) are frequently observed close to regions of steep topography in the ocean such as shelf-edges (Holloway et al., 1997) and ridges (Konyaev et al., 1995) and sills

* Corresponding author.
E-mail address: small@hawaii.edu (R.J. Small).

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(Morozov et al., 2002). ISWs typically occur in packets at tidal intervals, suggesting that they arise from non-linear transformation of large amplitude, longer wavelength internal tides (Gerkema, 1996). The generation of ISWs from the internal tide has received much attention (Lamb, 1994; Gerkema, 1996; Gerkema, 2001) but until recently the behaviour of ISWs as they cross a continental slope and shoal has been less studied.

Recent studies of the shoaling of ISWs have focused on laboratory experiments and fully non-linear computations (Vlasenko and Hutter, 2002; Sveen et al., 2002; Michallet and Ivey, 1999; Lamb, 2002). So far there has been no comparison of fully non-linear predictions of shoaling with weakly non-linear predictions. Further, few oceanic observations have been previously discussed. This paper aims to redress the issue and compare numerical and theoretical models with observations.

Although it is generally acknowledged that weakly non-linear theories will not adequately predict all the behaviour of large amplitude internal solitary waves, in some circumstances the theories are reasonably accurate (Small, 2003, hereafter S2003; Michallet and Barthelemy, 1998; Helfrich and Melville, 1986). One of the aims of this paper is to determine under what circumstances weakly non-linear theories will be appropriate, which will help in the choosing of appropriate models.

In a parallel study (S2003) the observed transformation of a non-linear internal wave packet at the Malin shelf and slope (reported in Small et al., 1999a,b) was compared against weakly non-linear models with no dissipation. A partially second order, variable-depth Extended Korteweg-de-Vries equation (vEKdV: Holloway et al., 1999) was found to reproduce the observed capping of wave amplitude as it moved across the continental slope. (In contrast the purely first order variable depth KdV (vKdV) equation predicted rapid wave growth to unrealistic amplitudes.) However, the capping may have been due to another mechanism, namely turbulent damping, that was not discussed in S2003.

In this present study we aim to determine the relative importance of laminar dynamics and turbulent processes to the non-linear internal wave evolution. To this end the shoaling of a single ISW is studied using the fully non-hydrostatic, non-linear computational fluid dynamics code PHOENICS (Hedberg et al., 1986), and the results are compared to the observations and vEKdV theory. The CFD model has a turbulence closure scheme (k–ε) which can be switched on or off to determine the relative importance of dissipation. It can therefore help to answer the question whether the amplitude capping over the slope is due to laminar non-linear dynamics (i.e. vEkdV predictions) or due to turbulent damping.

The paper is organised as follows. Section 2 describes the models to be used in this study. Section 3 contains the set up of the experiment and its motivation. Section 4 then contains the numerical experimental results, including details of wave evolution, and wave properties such as shape and phase speed, and compares the results to oceanographic observations. A discussion in Section 5 relates these results to previous findings. Finally conclusions are given in Section 6.

2. Experimental set up

In these simulations the propagation of ISWs across water depths from 400 to 140 m is considered. The experimental set up is chosen to allow comparison with the observations from the