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Efficient implementation of covariance multiplication for data assimilation with the representer method

Hans E. Ngodock

Department of Marine Science, University of Southern Mississippi, 1020 Balch Boulevard, Stennis Space Center, MS 39529, USA

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Abstract

The application of Markov time correlation within the representer method is revised. Usually, the Markov time correlation is implemented by solving a two-point boundary-value problem that is split into two Langevin equations, one forward and the other backward in time. A new splitting of the two-point boundary-value problem is proposed. An examination of time and memory consumption is carried out when portions of the trajectory are written and read from available disk space, and when checkpoints are used. It is shown that the new splitting will reduce the computation time and the core computer memory required to solve the variational data assimilation problem through the representer method.

Keywords: Representer method; Covariance multiplication; Checkpointing; Variational data assimilation

1. Introduction

To date, the covariance multiplication in the representer method is time and memory consuming, Bennett (2002, and personal communication), Purser et al. (2003). This may just be a matter of formulation. Can the covariance (particularly the time covariance) multiplication process in the representer method be reformulated to save some time and memory, and how much can be saved? The representer method for assimilating observations into numerical models is a

E-mail address: ngodock@nrlssc.navy.mil (H.E. Ngodock).

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weak constraint 4D-Var assimilation technique (e.g. Zupanski, 1997; Courtier, 1997). It produces a best estimate model solution that fits the data to within data error, and is able to estimate the model error sources (boundary conditions, forcing fields and/or bad parameterization) provided adequate statistical assumptions about those errors. The 4D-Var technique constructs a solution that minimizes errors to the dynamical equations and measurements over all space and time simultaneously. The solution is derived by minimizing a cost function that contains the misfits to the data and dynamical model, as well as the covariances specifying spatial and temporal correlations of errors. The weights in the cost function are the inverses of respective covariance functions. The representer method offers the advantage of not having to invert the covariance functions. Within the solution process, an adjoint model is first run forced by measurement functionals. The adjoint solution is multiplied by the covariance, and the forward model is forced by this result. While the adjoint and forward models may be expensive to run, the covariance multiplication will increase the computational cost.

However, the way these covariance functions are often applied to the adjoint solution is memory and time consuming. The current application of the covariance functions in the representer method requires the storage of the full adjoint trajectory; the adjoint equations are integrated backward over the assimilation time interval, kept in memory, and passed on to the covariance multiplication process where both the space and time covariance functions are applied to the adjoint solution or residuals. In practice, a simplifying assumption is applied. It is assumed that the model error covariance is separable in time and space, allowing a separate (yet sequential) correlation multiplication of the residuals, as one can apply the time correlation function to the residuals independently of the space correlation function (e.g. Chua and Bennett, 2001). It is the time covariance multiplication that would require the entire adjoint trajectory to be available in memory. In the particular case where the time covariance function is a Markov process, the covariance multiplication is done by solving a pair of Langevin equations: one forward in time (from the initial time to the final time of the integration), and the other backward. The spatial covariance functions may be applied either before or after the temporal covariance operation.

This study examines a new approach to handle the time covariance operation. It maintains the assumptions that the covariance is separable in space and time and that the time covariance is a Markov process. Thus the requirement to solve a pair of Langevin equations still holds, but the Langevin equations will be modified in order to avoid the storage of the entire adjoint trajectory. This allows the 4D-Var to perform the covariance multiplications in a way that would save computation memory and time. The numerical implementation of the Langevin equations may be likened to the application of recursive filters (Lorenc, 1992; Purser et al., 2003). In this study, the Langevin equations are discretized by a two-point forward Euler scheme, and their forward and backward integrations are similar to a one-pass recursive filter with carefully tuned coefficients. This paper is organized as follows: in Section 2 we review the representer method and typical covariance functions (Section 3) and how they are implemented within the representer method. In Section 4, we focus on the time covariance multiplication and introduce a new implementation. An examination of time and memory savings follows in Section 5 (when portions of the trajectory are written and read from available disk space), and when checkpoints are used (Section 6). Concluding remarks in Section 7 and Appendix A and B follow respectively.