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Modelling soil water retention scaling. Comparison of a classical fractal model with a piecewise approach

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Abstract

Soil water retention curves are very important for modeling and predicting geohydrological processes. Fractal-based models seem to be appropriate theoretical frameworks for investigating this soil characteristics. Since soil water retention is sensitive to both structure and soil texture, it is difficult to see a unique value of fractal dimension as a physical descriptor of both soil conditions. The objectives of this work were to (i) present a piecewise fractal approach to approximate the soil water retention data, (ii) test the model with previously published and unpublished data sets, and (iii) compare its performance with a traditional surface fractal model. The goodness-of-fit of the piecewise model was excellent (R^2>0.95). Almost all the soil water retention data (21 data sets in total) showed two fractal scaling regimes. The cutoff ranged from a minimum h_c=10.889 kPa (Ariana silty clay loam soil) to a maximum h_c=2951 kPa (Walla-Walla silt loam soil). The fractal dimension corresponding to the first domain ranged from D_{1p}=2.59 to D_{1p}=2.85, while those values corresponding to the second regime varied between D_{2p}=2.72 and D_{2p}=2.95. The fit of a classical surface fractal model rendered poor results in terms of goodness-of-fit parameters with a large dispersion of predicted water content values at low tensions. The presented piecewise approach could be consistent to some extent with the bimodal pore-size distributions usually observed in experimental studies.

Keywords: Soil water retention; Fractal dimension; Scaling laws

1. Introduction

Soil water retention curves are very important for modelling and predicting geohydrological processes in the vadose zone. Both closed-form (van Genuchten, 1980) and empirical models (Brooks and Corey, 1964; Visser, 1968; Campbell, 1974) have been proposed and used to parameterize the soil water retention. Any way, direct measurements of soil water retention are expensive and time-consuming. In fact, they are cost-effective only for site-specific problems (Wösten and van Genuchten, 1988).

The introduction of physical models based on the fractal geometry of natural media (Mandelbrot, 1982)

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provided soil scientists with the first integrated model of soil structure (Rieu and Sposito, 1991a). Estimation of fractal dimensions from soil water retention data is well documented in the literature (Rieu and Sposito, 1991b; Perfect et al., 1996; Perfect, 1999; Filgueira et al., 1999). These studies have assumed the existence of a unique fractal dimension (usually a mass fractal dimension) within the scale range where data are available, while other investigations have modelled the soil water retention as the effect of an underlying fractal surface (de Gennes, 1985). In reality, mass- and surface-based soil water retention models are similar in their mathematical formulation but their scaling exponents represent the mass fractal dimension, D_m, and the surface fractal dimension, D_s, respectively.

The mass-based models assume that soil pores desaturate as predicted by the Young–Laplace equation (Friesen and Mikula, 1987; Tyler and Wheatcraft, 1990), while surface-based models consider that water is present as a thin film (de Gennes, 1985; Toledo et al., 1990). In fact, we could be faced to contradictory results since, in general, $D_m > D_s$ for a given system. Several investigations have revealed that natural porous media (rocks, soils) can enclose more than one fractal regime (Avnir et al., 1985; Pachepsky et al., 1995, 1996; Bartoli et al., 1998). In addition to this information, there also exist solid evidences of an underlying bimodal distribution of soil pore space (structural and textural porosity) (Nimmo, 1997; Zhang and van Genuchten, 1994). Durner (1994) has pointed out that this bimodal character of pore-size distributions allows one a partitioning of the pore space when simulating the pore-size distribution. In the case of soil water retention, our main hypothesis is that both phenomena (capillary and adsorption) coexist, and each of then is dominant within a given scaling range generating its own fractal dimension. The objectives of this work were to (i) present a piecewise fractal approach to approximate the soil water retention data, (ii) test the model with previously published and unpublished data sets, and (iii) compare its performance with a traditional surface fractal model.

2. Theory

Perrier et al. (1999) developed a symmetric pore-solid fractal (PSF) model. Its main feature is the connection in the same geometric shape of a distribution of pores and a distribution of solids which both assume a power law function with the same fractal dimension (Perrier et al., 1999). Within the context of the PSF model, the soil water retention function assumes the form:

$$\theta(h) = \phi - \frac{x}{x+y}\left[1 - \left(\frac{h}{h_b}\right)^{D_p - 2}\right] \quad (1)$$

where θ is the volumetric water content, $\phi \approx \theta_s$ is the total soil porosity, x denotes the pore phase, y represents the solid phase, h_b is the tension draining the largest pore, and D_p is the fractal dimension. Bird et al. (2000) identified three special cases: when the solid phase, $y \rightarrow 0$, Eq. (1) becomes the mass fractal model developed by Rieu and Sposito (1991a), as the pore phase, $x \rightarrow 0$, Eq. (1) represents a step, nonfractal function, and as both, $x \neq 0$, and $y \neq 0$, $x/(x+y) \rightarrow \phi$, and Eq. (1) tends to the empirical models of Brooks and Corey (1964) and Campbell (1974), and the fractal model of de Gennes (1985):

$$\theta(h) = \phi\left(\frac{h}{h_b}\right)^{D_p - 3} \quad (2)$$

In the case of de Gennes (1985) model, D_p represents a surface fractal dimension.

Soil water retention is usually sensitive to soil structural and textural conditions. This means that more than one fractal domain, with different fractal parameters, could be present within most soils.

This also implies the existence of a critical value, h_c, separating two fractal regimes such that:

$$\theta(h) = \begin{cases} a_1 h^{D_{1p} - 3}, & h \leq h_c \\ a_2 h^{D_{2p} - 3}, & h > h_c \end{cases} \quad (3)$$

where D_{1p} and D_{2p} are considered here as associated to pore-size distributions (psds) corresponding to both domains. The h_c value partitions the whole domain into two scaling ranges, $h_b \leq h \leq h_c$, and $h_c \leq h \leq h_{max}$. The composite scaling constant, a_1, can be presented as:

$$a_1 = \theta_s h_b^{3 - D_{1p}} \quad (4)$$

while the constant a_2 can be approximated as:

$$a_2 = \theta_c h_c^{3 - D_{2p}} \quad (5)$$