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Prediction boundaries and forecasting of non linear hydrologic stage data

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Abstract

This paper analyzes and forecasts hydrologic stage data in the Loxahatchee National Wildlife Refuge, the northernmost extent of the Florida Everglades. Analysis indicates that the process dynamics are chaotic, for which several attractor invariants are evaluated. A connection is made between the Kolmogorov-Sinai (KS) entropy of the phase-space trajectories and limits of temporal stage predictability. Evaluation of the KS entropy establishes a boundary for forecast time periods. Comparisons are presented between inferences produced by linear statistical models and the nonlinear attractor invariants. A nonlinear estimator (feedforward neural network), along with several linear models are employed to perform temporal forecasts of stage data. The observed degradation of forecast accuracy is consistent with the limits inferred from the attractor entropy. The nonlinear estimator is found to have better prediction accuracy than the linear models for prediction intervals beyond several days.

Keywords: Nonlinear dynamics; Stage prediction; Artificial neural network

1. Introduction

Many applications of analytical scrutiny applied to real-world, physical systems are based on assumptions of linearity. These assumptions are often taken for granted, and may lead to inaccuracies as a result of applying linear modeling and forecasting techniques to physical systems that exhibit nonlinear dynamics. Nonlinear dynamics are nherent in chaotic systems: multivariate processes that exhibit internal order, long term stability, sensitive dependence on initial conditions and significant dynamic variations. While the process dynamics are stable in a global sense, the local phase-space trajectories of chaotic system variables are inherently unpredictable as forecast time scale increases. Chaotic systems maintain exquisite balances between the nonlinear forces of dissipation, randomness and internal order, and therefore, are often not best analyzed in the linear domain. It is notable that such realizations evolved from the celebrated work of Lorenz (Lorenz, 1963) who was concerned with forecasting atmospheric flows. Subsequently, evidence of chaotic behavior in hydrological time-series has been found since the late

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1980s. Even though much debate ensued over the veracity of some of these findings, it is now well accepted that low-dimensional determinism is indeed a feature of many hydrological processes (Sivakumar, 2000, 2004). In addition to the detection of chaotic behavior in hydrological processes, application of nonlinear forecasting methods such as artificial neural networks (ANN) (Elshorbagy et al., 2002a,b; Sivakumar et al., 2002; Lambrakis, 2000), k-nearest neighbors (Elshorbagy et al., 2002a), and phase-space reconstruction (Sivakumar et al., 2002) have resulted in viable nonlinear forecasting procedures. The majority of these analysis have focused on rainfall or stream flow dynamics (Sivakumar, 2000, 2004), while to the authors knowledge, few have addressed waterbody volume/stage dynamics (Sangoyomi et al., 1996).

This study applies the methods of nonlinear dynamical systems theory to the analysis of daily hydrologic stage variations in the Loxahatchee National Wildlife Refuge. It is suggested that the process dynamics are chaotic; meaning that the phase-space attractor has a fractal dimension, a positive Lyapunov exponent, and occupies a finite domain. These features imply that linear modeling of the stage evolution could lead to inaccuracies in forecasting. Quantification of several attractor invariants leads to an estimate of the dimensionality of the dynamics, as well as a measure of the local trajectory divergence. Through connection of the trajectory divergence rate and the information capacity of the attractor in terms of the observed state values of the dynamics, a limit on the predictability of fine-scale stage forecasting is established. Comparisons to the predictability inferred from linear correlations are presented.

Having established some properties of the nonlinear dynamics, the study turns to application of a nonlinear model applied to forecasting the stage data. An artificial neural network is constructed to learn, and predict stage evolution within the boundaries of the identified predictability limits. The neural network is employed to forecast stages for future days over a 16 year time period, based on two years of training data. Additionally, several linear models are constructed and their forecasting results compared to those of the ANN. Errors of the forecasts are examined with respect to the predictability derived from analysis of the attractor dynamics.

To provide a basis for subsequent discussion on the features of linear and nonlinear modeling, a brief review of the two approaches is outlined below.

1.1. Linear models

Consider a linear system with f degrees of freedom where the time evolution of each component is described by the vector $\boldsymbol{\mu}(t)=[\mu_1(t),\mu_2(t),\ldots,\mu_f(t)]$ so that

$$\frac{\mathrm{d}\boldsymbol{\mu}(t)}{\mathrm{d}t}=\mathbf{A}\cdot\boldsymbol{\mu}(t) \tag{1}$$

where $\mathbf{A}$ is a constant $f\times f$ matrix. Solution of this equation specifies trajectories of $\boldsymbol{\mu}(t)$ in f-space for which one of three outcomes is possible:

1. Real part of the eigenvalues of $\mathbf{A}$ are positive.
2. Real part of the eigenvalues of $\mathbf{A}$ are negative.
3. Eigenvalues of $\mathbf{A}$ occur in complex-conjugate pairs with either zero or negative real parts.

In the case of negative eigenvalues, the trajectories of $\boldsymbol{\mu}(t)$ eventually collapse to a stable point, such as in the case of a pendulum subjected to friction. If only positive eigenvalues are present the orbits of $\boldsymbol{\mu}(t)$ grow to infinity, a situation which clearly invalidates a stable physical model. The occurrence of positive eigenvalues ensures that the assumption of linear dynamics is incorrect, in which case one may employ nonlinear evolution equations to better approximate the dynamics.

In most modeling applications one has access to sampled versions of the process dynamics at a fixed spatial point

$$s(n)=s(t_0+n\tau_s) \tag{2}$$

where t_0 is the initial time and τ_s the sampling interval for the nth observation. In linear analysis, it is assumed that the observations are linearly related to previous observations and forcing terms

$$s(n)=\sum_{i=1}^{N}a_i s(n-i)+\sum_{j=1}^{M}b_j\phi(n-j) \tag{3}$$

where the a_i and b_j are model coefficients and ϕ represent the forcing terms. This integral representation is typically cast into an algebraic form through