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Parameter estimation for leaky aquifers using the extended Kalman filter, and considering model and data measurement uncertainties

H.D. Yeh*, Y.C. Huang

Institute of Environmental Engineering, National Chiao-Tung University, 75 Po-Ai Street, Hsinchu 30039, Taiwan

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Abstract

A method using the extended Kalman filter (EKF) is proposed to identify the hydraulic parameters in leaky aquifer systems both with and without considering the aquitard storage. In the case without considering the aquitard storage, Hantush and Jacob's model combined with EKF can optimally determine the parameters for the leaky aquifer when analyzing the drawdown data. Coupled with Neuman and Witherspoon's model, the EKF is also employed to estimate the four parameters of aquifers. The observed drawdown data may be either interpolated using the Lagrangian polynomial or recursively used while implementing the EKF. The proposed method can identify the parameters, using part of the interpolated drawdown data or recursively used data, and obtains results with good accuracy. In the field-pumping test, a long pumping time may not be necessary if the proposed method is implemented on a computer, which is connected to pressure transducers and a data logger. In the process of parameter estimation, the leakage coefficient changes marginally for the first few observations. This phenomenon reflects the fact that there is a time lag between the start of pumping and the leakage effect on the drawdown. The analyses of the data uncertainty demonstrate that the EKF approach is applicable for drawdown data even when it contains white noise or temporal correlated noise. Finally, the choice between Hantush and Jacob's model and Neuman and Witherspoon's model depends on the hydrogeological condition of the aquifer system indicated in the analyses of the model uncertainty. Hantush and Jacob's model is shown to be a good choice for representing the leaky aquifer system if the aquitard storage is comparatively small.

Keywords: Parameter estimation; Kalman filter; Lagrangian polynomial; Groundwater; Leaky aquifer; Model uncertainty

1. Introduction

Hydrogeologic parameters are very important in site characterization, so groundwater hydrologists often conduct pumping tests to determine hydrogeologic parameters, such as hydraulic conductivity and storage coefficient. These parameters are necessary information for quantitative and/or qualitative groundwater studies. In a leaky aquifer the semi-pervious bed (also known as the aquitard), although of very low permeability, may yield significant amounts of water to the adjacent pumped aquifer. As time

* Corresponding author. Fax: +886-35-726050.
E-mail address: hdyeh@mail.nctu.edu.tw (H.D. Yeh).

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increased, leakage across the semi-impervious bed may become appreciable and flow is not restricted to the pumped aquifer alone. The additional water may be derived from storage of the aquitard and adjacent unpumped aquifers. Therefore, the leaky aquifer must be viewed as part of a complex multiple aquifer system.

Two approaches have been developed for dealing with leaky aquifers, one based on the assumption that the aquitard storage is negligible, and the other considering the aquitard storage. Jacob (1946) developed a partial differential equation for non-steady radial flow in a leaky aquifer, assuming that hydraulic head in the unpumped adjacent aquifer is constant and the storage capacity of the aquitard is negligible. These two assumptions greatly facilitated mathematical treatment of the problem, and Jacob used this approach to develop a large number of solutions to various problems involving flow in aquifers with vertical leakage.

Hantush and Jacob (1955) described non-steady radial flow to a well in a fully penetrated leaky aquifer under a constant pumping rate. In their model, the aquitard is overlain by an unconfined aquifer, and the main aquifer is underlain by an impermeable bed. Their solution is herein called the three-parameter model. Hantush (1960) also presented a modified approach to include the effect of the aquitard storage. Neuman and Witherspoon (1969) gave a solution describing the drawdown of the lower and pumped aquifer in a hydrogeologic system, which is composed of two confined aquifers and one aquitard. Their solution, which considers the effect of aquitard storage and neglects the drawdown in the unpumped aquifer, is called the four-parameter model. Both the three-parameter and four-parameter models are also mentioned in several recent textbooks, for example, Dawson and Istok (1991) and Batu (1998).

In the three-parameter model, the graphical method based on Hantush's or Walton's type curves (Batu, 1998) requires data plotting work and individual judgment during the curve fitting procedure. Therefore, errors may be introduced during the fitting process. In the four-parameter model, the use of the graphical matching method based on the Neuman and Witherspoon's model is practically impossible since there will be several families of type curves. Yeh (1987) used the non-linear least-squares and finite-difference Newton's method (NLN) for identifying the parameters of the confined aquifer, a method which has the advantages of high accuracy and quick convergence. Yeh and Han (1989) subsequently used NLN to determine the hydraulic parameters of the leaky aquifers.

The Kalman filter was developed by R.E. Kalman in the late 1950s, and its main applications have been in control systems, tracking and navigation of all sorts of vehicles, as well as predictive design of estimation and control systems. Works using the Kalman filter for hydraulic-parameter and water table-related estimations may be divided into two categories. One applies the Kalman filter in a linear system (e.g. Van Geer and Van Der Kloet, 1985; Van Geer and te Stroet, 1990; Van Geer et al., 1990; Lee et al., 2000; Bierkens et al., 2001) and the other deals with non-linear problems using the extended Kalman filter (EKF) (e.g. Chander et al., 1981; Katul et al., 1993; Bierkens, 1998; Cahill et al., 1999).

Chander et al. (1981) estimated the parameters for both non-leaky and leaky aquifers by the iterated EKF. For the leaky aquifer, the measurement equation was a truncated form of Hantush and Jacob's well function (1955), which was suitable for small leakage coefficient and/or large pumping time. Van Geer and Van Der Kloet (1985) presented two linear, filter-based schemes for parameter estimation in groundwater flow problems. An optimal estimate was simultaneously computed for the original state, i.e., heads and the parameter state. Van Geer and te Stroet (1990) combined MODFLOW with a filtering framework and updated the prior estimates of hydraulic parameter values using an off-line procedure, when minimizing the difference between the actual head measurements and those predicted from the MODFLOW–Kalman filter framework. Van Geer et al. (1990) also adopted the idea of using a filter for state estimation in the absence of significant dynamic behavior and studied the applicability of the filter to a relatively quickly reacting groundwater system.

Katul et al. (1993) applied the EKF to test the determination of the hydraulic conductivity function from a field drainage experiment. Bierkens (1998) embedded the stochastic differential equation in the EKF algorithm to calibrate the parameters and noise statistics of the stochastic differential equation on a time series of water table depths. Eigbe et al. (1998)