



Implicit function theorem over free groups

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Abstract

We introduce the notion of a regular quadratic equation and a regular NTQ system over a free group. We prove the results that can be described as implicit function theorems for algebraic varieties corresponding to regular quadratic and NTQ systems. We will also show that the implicit function theorem is true only for these varieties. In algebraic geometry such results would be described as lifting solutions of equations into generic points. From the model theoretic view-point we claim the existence of simple Skolem functions for particular $\forall\exists$ -formulas over free groups. Proving these theorems we describe in details a new version of the Makanin–Razborov process for solving equations in free groups. We also prove a weak version of the implicit function theorem for NTQ systems which is one of the key results in the solution of the Tarski’s problems about the elementary theory of a free group. We call it the parametrization theorem.

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1. Introduction

In this paper we prove so-called implicit function theorems for regular quadratic and NTQ systems over free groups (Theorems 3, 9, 11). They can be viewed as analogs of the corresponding result from analysis, hence the name. To show this we formulate a very basic version of the implicit function theorem.

Let

$$S(x_1, \dots, x_n, a_1, \dots, a_k) = 1$$

be a “regular” quadratic equation in variables $X = (x_1, \dots, x_n)$ with constants a_1, \dots, a_k in a free group F (roughly speaking “regular” means that the radical of S coincides with the normal closure of S and S is not an equation of one of few very specific types). Suppose

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