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# Radical 2-subgroups of the Monster and the Baby Monster

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**Abstract**

The radical 2-subgroups of the Monster and the Baby Monster are determined up to conjugacy, which completes the classification problem of radical 2-subgroups for all sporadic simple groups.


## Introduction

Let $p$ be a prime dividing the order of a finite group $G$. A nontrivial $p$-subgroup $R$ of $G$ is called *radical* if $R = O_p(N_G(R))$, where $O_p(X)$ denotes the largest normal $p$-subgroup of a group $X$. In this paper, the radical 2-subgroups of the Monster and the Baby Monster are determined up to conjugacy, based on the remarkable classification by Meierfrankenfeld and Shpectorov of maximal 2-local subgroups of these groups [14,15]. Together with previous works, this completes the classification of radical 2-subgroups of all sporadic simple groups. To emphasize the completeness, literature for each sporadic group is given in the appendix, though the list may not be comprehensive. There are included the lists of radical 2-subgroups for $J_2$ and $McL$, as we are unaware of any published works for them.

See the introductions of [22] and [13] for the motivations and the strategy to determine the radical $p$-subgroups. Note that classification of the radical subgroups is required to

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verify Dade's conjecture. For the Monster and the Baby Monster, their maximal 2-local subgroups are neatly described in [15] and [14], which allows us to carry out the classification of their radical 2-subgroups rather smoothly. We need some fusion arguments to examine the candidates for radical 2-subgroups whose normalizers are contained in a maximal 2-local subgroup in the Baby Monster which is not of "characteristic-2" type. Except this point (see Section 1.2), all the information we need can be found in [15] and [14]. In that sense, this paper is just a corollary of their remarkable classification.

The main results are summarized in Tables 1 and 2. There, for a representative $R$ of each class of the radical 2-subgroups, we give a brief description of structure of $R$ as well as those of the center $Z(R)$ and the automizer $N_G(R)/R$. Following [15], we use the symbol $\sim$ (e.g., $L_3 \sim 2_+^{1+24}Co_1$) as shorthand for "has structure," while $\cong$ as usual stands for "is isomorphic." To describe the structure of a group, we follow the Atlas notation, in particular, $A.B$ (or $AB$) means an extension of $B$ by $A$ (a group with a normal subgroup isomorphic to $A$ and the factor group isomorphic to $B$). Classes of radical subgroups with isomorphic centers are arranged in the tables according to the fusion of the elements in the center, with class names following the Atlas notation. If fusion is not indicated, the center is identical with the group in the previous row.

Recall that a radical $p$-subgroup $R$ is called *centric*, if any $p$-element centralizing $R$ lies in $Z(R)$. For the importance of centric radical subgroups, see [19] and [20, §4]. We also refer to which representatives are centric, without giving any verification, as it is easy to see.

The symbols $S_n$ and $A_n$ are used to denote the symmetric and alternating groups of degree $n$. In Table 2, the direct product $S_3 \times S_3$ and $S_3 \times S_3 \times S_3$ are abbreviated to $S_3^2$ and $S_3^3$. We also use the symbols $n$, $p^n$ and $2_\varepsilon^{1+2n}$ to denote, respectively, the cyclic group of order $n$, the elementary abelian group of order $p^n$ and the extraspecial group of order $2^{1+2n}$ of $\varepsilon$ type ($\varepsilon = \pm 1$). Furthermore, the symbols $D_8$, $Q_8 \cong 2_-^{1+2}$ and $SD_{16}$ indicate the dihedral, quaternion and semidihedral group of order 8, 8 and 16, respectively.

For a conjugacy class $pX$ of an element of order $p$, an elementary abelian $p$-subgroup is called *$pX$-pure* if all its nontrivial elements lie in $pX$. I also call a subgroup $H$ of $G$ a *$X$-subgroup*, if $H \cong X$.

## 1. Radical 2-subgroups of the Baby Monster

### *1.1. 2-local subgroups of BM*

There are four conjugacy classes of involutions in $B := BM$, the Baby Monster, called $2A$, $2B$, $2C$ and $2D$, with centralizers of shapes $2 \cdot {}^2E_6(2).2$, $2_+^{1+22} \cdot Co_2$, $(2^2 \times F_4(2)).2$, and $2^9.2^{16}\Omega_8^+(2).2$, respectively. It follows from [15, Theorem 2] and [14, Theorem B] that every 2-local subgroup of $B$ is contained in one of the following eight subgroups up to conjugacy:

$L_1 \sim 2 \cdot {}^2E_6(2).2$;
$L_2 \sim (2^2 \times F_4(2)).2$;