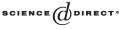


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## Schilder theorem for the Brownian motion on the diffeomorphism group of the circle

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## Abstract

We prove a large deviation principle for flows associated to stochastic differential equations with non-Lipschitz coefficients. As an application we establish a Schilder Theorem for the Brownian motion on the group of diffeomorphisms of the circle. © 2004 Elsevier Inc. All rights reserved.

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## 1. Introduction

This paper is a continuation of our previous work [17]. Our motivation is to prove a Schilder theorem on the asymptotic behavior for the Brownian motion on the group of homeomorphisms of the circle which is constructed by Malliavin [16].

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Recall that the original Schilder theorem [18] states that if *B* is the real Brownian motion and  $C_0[0, 1]$  the space of real continuous functions defined on [0,1], null at 0, and endowed with the uniform convergence norm, then for any Borel set  $A \subset C_0[0, 1]$ ,

$$\begin{split} \limsup_{\varepsilon \to 0} \varepsilon^2 \log P\{\varepsilon B \in A\} \leqslant -\Lambda(\bar{A}), \\ \liminf_{\varepsilon \to 0} \varepsilon^2 \log P\{\varepsilon B \in A\} \geqslant -\Lambda(A^0), \end{split}$$

where  $\bar{A}$  and  $A^0$  denote the closure and interior of A respectively, and  $\Lambda(A) := \inf_{\gamma \in A} \lambda(\gamma)$  with

$$\lambda(\gamma) = \begin{cases} \frac{1}{2} \int_0^1 |\dot{\gamma}(s)|^2 \, ds, & \gamma \text{ absolutely continuous,} \\ \infty, & \text{otherwise.} \end{cases}$$
(1)

This result was then generalized by Freidlin and Wentzell in their famous paper [10] by considering the Itô equation

$$\begin{cases} dX^{\varepsilon}(t) = \varepsilon \sigma(X^{\varepsilon}(t)) \, dw_t + b(X^{\varepsilon}(t)) \, dt & 0 \leq t \leq 1, \\ X^{\varepsilon}(0) = x. \end{cases}$$
(2)

They proved, under broad conditions (mainly the Lipschitzness and the boundedness of the coefficients), that for every Borel set A of  $C_x([0, 1], \mathbb{R}^d)$ ,

$$-\Lambda(A^0) \leqslant \liminf_{\varepsilon \to 0} \varepsilon^2 \log P(X_x^{\varepsilon} \in A) \leqslant \limsup_{\varepsilon \to 0} \varepsilon^2 \log P(X_x^{\varepsilon} \in A) \leqslant \Lambda(\bar{A}),$$

where  $C_x([0, 1], \mathbb{R}^d)$  is the space of continuous functions defined on [0, 1] and valued in  $\mathbb{R}^d$ , null at 0. Here also, the closure and the interior are taken in the topology of uniform convergence.

Changing the angle we can let x run over  $\mathbb{R}^d$  and regard the solution as a random variable taking values in  $C([0, 1], D(\mathbb{R}^d))$  where  $D(\mathbb{R}^d)$  is the space of  $C^{\infty}$  functions on  $\mathbb{R}^d$  endowed with the topology of compact uniform convergence of derivatives of all orders, provided that the coefficients are smooth and of linear growth (see [12,14,15]). Thus one can also consider the large deviation principle (LDP in abbreviation) in this context, namely the asymptotic estimates of probabilities  $P(X^{\varepsilon} \in A)$  where  $A \in \mathcal{B}(C([0, 1], D(\mathbb{R}^d)))$ . See e.g. [2,11] among others for studies along this line.

Now let us return to the Brownian motion on the group of homeomorphisms of the circle. It is defined by a SDE as in (20) below. The main features of this equation are

- (i) The coefficients are not Lipschitz, not to mention differentiable;
- (ii) Infinitely many driving Brownian motions are involved.

To deal with such an equation we will first work in a more general context by considering flows generated by SDE in  $\mathbb{R}^d$  with non-Lipschitz coefficients and driven

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