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On the topology of the Kasparov groups and its applications

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Abstract

In this paper we establish a direct connection between stable approximate unitary equivalence for $*$-homomorphisms and the topology of the KK-groups which avoids entirely C^*-algebra extension theory and does not require nuclearity assumptions. To this purpose we show that a topology on the Kasparov groups can be defined in terms of approximate unitary equivalence for Cuntz pairs and that this topology coincides with both Pimsner's topology and the Brown–Salinas topology. We study the generalized Rørdam group $KL(A, B) = KK(A, B)/\bar{0}$, and prove that if a separable exact residually finite dimensional C^*-algebra satisfies the universal coefficient theorem in KK-theory, then it embeds in the UHF algebra of type 2^∞. In particular such an embedding exists for the C^*-algebra of a second countable amenable locally compact maximally almost periodic group.
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1. Introduction

Two $*$-homomorphisms $\varphi, \psi : A \to B$ are unitarily equivalent if $u\varphi u^* = \psi$ for some unitary $u \in B$. They are approximately unitarily equivalent, written $\varphi \approx_u \psi$, if there is

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a sequence $(u_n)_{n\in\mathbb{N}}$ of unitaries in B such that

$$\lim_{n\to\infty} \|u_n\varphi(a)u_n^* - \psi(a)\| = 0$$

for all $a \in A$. Stable approximate unitary equivalence is a more elaborated concept introduced in Definition 3.6. According to Glimm's theorem, any non-type I separable C^*-algebra has uncountably many non-unitarily equivalent irreducible representations with the same kernel. In contrast, by Voiculescu's theorem, two irreducible representations of a separable C^*-algebra have the same kernel if and only if they are approximately unitarily equivalent. A comparison of the above results suggests that the notion of unitary equivalence is sometimes too rigid and that for certain purposes one can do more things by working with approximate unitary equivalence. This point of view is illustrated by Elliott's intertwining argument: if $\varphi : A \to B$ and $\psi : B \to A$ are unital $*$-homomorphisms between separable C^*-algebras such that $\varphi\psi \approx_u id_B$ and $\psi\varphi \approx_u id_A$, then A is isomorphic to B. It is therefore very natural to study approximate unitary equivalence of $*$-homomorphisms in a general context.

Two approximately unitarily equivalent $*$-homomorphisms $\varphi, \psi : A \to B$ induce the same map on K-theory with coefficients, but they may have different KK-theory classes. In order to handle this situation, Rørdam introduced the group $KL(A, B)$ as the quotient of $\mathrm{Ext}(SA, B)^{-1} \cong KK(A, B)$ by the subgroup $\mathrm{PExt}(K_{*-1}(A), K_*(B))$ of $\mathrm{Ext}(K_{*-1}(A), K_*(B))$ generated by pure group extensions [25]. This required the assumption that A satisfies the universal coefficient theorem (UCT) of [27]. Using a mapping cylinder construction, Rørdam showed that two approximately unitarily equivalent $*$-homomorphisms have the same class in $KL(A, B)$. On the other hand, a topology on the Ext-theory groups was considered by Brown–Douglas–Fillmore [4], and shown to have interesting applications in [3,28]. This topology, called hereafter the Brown–Salinas topology, is defined via approximate unitary equivalence of extensions. It was further investigated by Schochet in [31,32] and by the author in [7]. Schochet showed that the Kasparov product is continuous with respect to the Brown–Salinas topology for K-nuclear separable C^*-algebras. An important idea from [31,32] is that one can use the continuity of the Kasparov product in order to transfer structural properties between KK-equivalent C^*-algebras. As it turns out, the subgroup $\mathrm{PExt}(K_{*-1}(A), K_*(B))$ of $\mathrm{Ext}(SA, B)^{-1}$ coincides with the closure of zero in the Brown–Salinas topology under the assumption that A is nuclear and satisfies the UCT. It is then quite natural to define $KL(A, B)$ for arbitrary separable C^*-algebras as $\mathrm{Ext}(SA, B)^{-1}/\bar{0}$ as proposed by Lin in [20]. Nevertheless, the study of $*$-homomorphisms from A to B via their class in $\mathrm{Ext}(SA, B)^{-1}$ is not optimal and leads to rather involved arguments as those in [19,20,7] where the Brown–Salinas topology of $\mathrm{Ext}(SA, B)^{-1}$ is related, in the nuclear case, to stable approximate unitary equivalence of $*$-homomorphisms from A to B.

Kasparov's KK-theory admits several equivalent descriptions. This deep feature enables one to choose working with the picture that is most effective in a given situation. Similarly, there are several (and as we are going to see, equivalent) ways to introduce a topology on the KK-groups. The Brown–Salinas topology was already

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