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On diffusion in high-dimensional Hamiltonian systems

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Abstract

The purpose of this paper is to construct examples of diffusion for ε-Hamiltonian perturbations of completely integrable Hamiltonian systems in $2d$-dimensional phase space, with d large.

In the first part of the paper, simple and explicit examples are constructed illustrating absence of 'long-time' stability for size ε Hamiltonian perturbations of quasi-convex integrable systems already when the dimension $2d$ of phase space becomes as large as $\log \frac{1}{\varepsilon}$. We first produce the example in Gevrey class and then a real analytic one, with some additional work.

In the second part, we consider again ε-Hamiltonian perturbations of completely integrable Hamiltonian system in $2d$-dimensional space with ε-small but not too small, $|\varepsilon| > \exp(-d)$, with d the number of degrees of freedom assumed large. It is shown that for a class of analytic time-periodic perturbations, there exist linearly diffusing trajectories. The underlying idea for both examples is similar and consists in coupling a fixed degree of freedom with a large number of them. The procedure and analytical details are however significantly different. As mentioned, the construction in Part I is totally elementary while Part II is more involved, relying in particular on the theory of normally hyperbolic invariant manifolds, methods of generating functions, Aubry–Mather theory, and Mather's variational methods.

Part I is due to Bourgain and Part II due to Kaloshin.
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Part I: an example of diffusion for Hamiltonian perturbations of integrable systems in high dimension

1. Introduction

Consider a real analytic Hamiltonian, expressed in action-angle variables, of the form

$$H(I, \theta) = h(I) + \varepsilon f(I, \theta), \tag{1.1}$$

where $(I, \theta) \in \mathbb{R}^d \times \mathbb{T}^d$ and h, f are real analytic.

The corresponding equations of motion are

$$\begin{cases} \dot{I}_j = -\frac{\partial H}{\partial \theta_j} = -\varepsilon \frac{\partial f}{\partial \theta_j} \\ \dot{\theta}_j = \frac{\partial H}{\partial I_j} = \frac{\partial h}{\partial I_j} + \varepsilon \frac{\partial f}{\partial I_j} \end{cases} \quad (1 \leqslant j \leqslant d). \tag{1.2}$$

Thus H is an ε-perturbation of the integrable Hamiltonian $h(I)$, which we assume moreover to satisfy a strict convexity or quasi-convexity property. Recall that quasi-convexity means that

$$\langle D^2 h(I) v, v \rangle \geqslant c|v|^2 \tag{1.3}$$

required only to hold for vectors v orthogonal to $\nabla h(I)$.

A typical example of a quasi-convex h is

$$h(I) = I_1^2 + \cdots + I_d^2 + I_{d+1}.$$

The interest of the weaker quasi-convexity assumption is that it allows non-autonomous perturbations with a time-periodic dependence of a strictly convex Hamiltonian. Thus

$$H(I, \theta, t) = \sum_{j=1}^{d} I_j^2 + \varepsilon f(I, \theta, t) \tag{1.4}$$

with f 1-periodic in t, may be put in the quasi-convex format, considering an extra pair of action-angle variables (I_{d+1}, θ_{d+1}) and putting

$$\begin{aligned} &\mathcal{H}(I_1, \ldots, I_d, I_{d+1}; \theta_1, \ldots, \theta_d, \theta_{d+1}) \\ &\quad = \sum_{j=1}^{d} I_j^2 + I_{d+1} + \varepsilon f(I_1, \ldots, I_d; \theta_1, \cdots, \theta_d, \theta_{d+1}). \end{aligned} \tag{1.5}$$

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