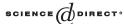


Available online at www.sciencedirect.com



Journal of Functional Analysis 229 (2005) 1-61

JOURNAL OF Functional Analysis

www.elsevier.com/locate/jfa

On diffusion in high-dimensional Hamiltonian systems

Jean Bourgain^{a,*}, Vadim Kaloshin^b

^aInstitute for Advanced Study, Princeton, NJ 08540, USA ^bCalifornia Institute of Technology, Pasadena, CA 91125, USA

> Accepted 30 September 2004 Communicated by J. Bourgain Available online 13 December 2004

Abstract

The purpose of this paper is to construct examples of diffusion for ε -Hamiltonian perturbations of completely integrable Hamiltonian systems in 2d-dimensional phase space, with d large.

In the first part of the paper, simple and explicit examples are constructed illustrating absence of 'long-time' stability for size ε Hamiltonian perturbations of quasi-convex integrable systems already when the dimension 2d of phase space becomes as large as $\log \frac{1}{\varepsilon}$. We first produce the example in Gevrey class and then a real analytic one, with some additional work.

In the second part, we consider again ε -Hamiltonian perturbations of completely integrable Hamiltonian system in 2d-dimensional space with ε -small but not too small, $|\varepsilon| > \exp(-d)$, with d the number of degrees of freedom assumed large. It is shown that for a class of analytic time-periodic perturbations, there exist linearly diffusing trajectories. The underlying idea for both examples is similar and consists in coupling a fixed degree of freedom with a large number of them. The procedure and analytical details are however significantly different. As mentioned, the construction in Part I is totally elementary while Part II is more involved, relying in particular on the theory of normally hyperbolic invariant manifolds, methods of generating functions, Aubry–Mather theory, and Mather's variational methods.

Part I is due to Bourgain and Part II due to Kaloshin.

© 2004 Elsevier Inc. All rights reserved.

E-mail address: bourgain@math.ias.edu (J. Bourgain).

^{*} Corresponding author.

Part I: an example of diffusion for Hamiltonian perturbations of integrable systems in high dimension

1. Introduction

Consider a real analytic Hamiltonian, expressed in action-angle variables, of the form

$$H(I,\theta) = h(I) + \varepsilon f(I,\theta), \tag{1.1}$$

where $(I, \theta) \in \mathbb{R}^d \times \mathbb{T}^d$ and h, f are real analytic.

The corresponding equations of motion are

$$\begin{cases} \dot{I}_{j} = -\frac{\partial H}{\partial \theta_{j}} = -\varepsilon \frac{\partial f}{\partial \theta_{j}} \\ \dot{\theta}_{j} = \frac{\partial H}{\partial I_{i}} = \frac{\partial h}{\partial I_{j}} + \varepsilon \frac{\partial f}{\partial I_{j}} \end{cases}$$
 (1 \leq j \leq d). (1.2)

Thus H is an ε -perturbation of the integrable Hamiltonian h(I), which we assume moreover to satisfy a strict convexity or quasi-convexity property. Recall that quasi-convexity means that

$$\langle D^2 h(I)v, v \rangle \geqslant c|v|^2 \tag{1.3}$$

required only to hold for vectors v orthogonal to $\nabla h(I)$.

A typical example of a quasi-convex h is

$$h(I) = I_1^2 + \dots + I_d^2 + I_{d+1}.$$

The interest of the weaker quasi-convexity assumption is that it allows non-autonomous perturbations with a time-periodic dependence of a strictly convex Hamiltonian. Thus

$$H(I, \theta, t) = \sum_{j=1}^{d} I_j^2 + \varepsilon f(I, \theta, t)$$
(1.4)

with f 1-periodic in t, may be put in the quasi-convex format, considering an extra pair of action-angle variables (I_{d+1}, θ_{d+1}) and putting

$$\mathcal{H}(I_{1}, \dots, I_{d}, I_{d+1}; \theta_{1}, \dots, \theta_{d}, \theta_{d+1})$$

$$= \sum_{j=1}^{d} I_{j}^{2} + I_{d+1} + \varepsilon f(I_{1}, \dots, I_{d}; \theta_{1}, \dots, \theta_{d}, \theta_{d+1}). \tag{1.5}$$

Download English Version:

https://daneshyari.com/en/article/9495831

Download Persian Version:

https://daneshyari.com/article/9495831

Daneshyari.com