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Θ-correspondences $(U(1), U(2))$, Part II: ramified case

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Abstract

We find an explicit decomposition for the oscillator (Weil) representation restricted to either member of the reductive dual pair $(U(1), U(2))$ in $\widetilde{Sp}(4, F)$ where F is a p-adic field with p odd.

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1. Introduction and notation

1.1. Introduction

Let F be a local *p-adic* field of odd residual characteristic. Let D be the quaternion division algebra over F. In our previous paper, "θ-Correspondences attached to $(U(1), U(2))$: Unramified Case", we parametrized the theta correspondence for the reductive dual pair $(U(1), U(2))$, where $U(1) = E^1$ is the norm one elements group of E, the unramified quadratic extension of F contained in D. In this paper we consider the reductive dual pair $(U(1), U(2))$ and parametrize the theta correspondence when E is a *ramified* quadratic extension of F contained in D. In the unramified case we

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used lattice model to parametrize the theta correspondence. Here we will be using a self-dual lattice model to parametrize the theta correspondence. Our main results in this case, when E/F is ramified are accumulated in the following theorems.

Let $\alpha \in D^{\circ}$ with $v_D(\alpha) = -n-1$ with n a positive integer, and $r = \left[\frac{n+1}{2}\right]$, where $[\,]$ is the greatest integer part function. Let χ_α be a character of D_r^1. Then there exists a character φ of L^1 such that $\varphi_{|L^1 \cap D_r^1} = \chi_{\alpha|L^1 \cap D_r^1}$, where L^1 is norm one elements group of $L = F(\alpha)$. For this φ define

$$\varphi_\alpha : L^1 D_r^1 \to \mathbb{C}^\times,$$

$$\varphi_\alpha\left(e\delta\right) = \varphi(e)\,\chi_\alpha\left(\delta\right).$$

Then φ_α is a character of $L^1 D_r^1$. Now for any character of $E^1(E^1_{\left[\frac{r+1}{2}\right]})$, ξ, define $\varphi_{(\alpha,\xi)}$, on $St\left(\chi_\alpha\right)$, the stabilizer in $U(2)$ of χ_α by

$$\varphi_{(\alpha,\xi)} : St\left(\chi_\alpha\right) \to \mathbb{C}^\times,$$

$$\varphi_{(\alpha,\xi)}\left(x,\lambda\right) = \varphi_\alpha(x)\,\xi\left(\lambda\right).$$

Then $\varphi_{(\alpha,\xi)}$ is a character [15]. Now let:

$$\rho_{(\alpha,\varphi,\xi)} = Ind.\left(U(2)\,, St\left(\chi_\alpha\right), \varphi_{(\alpha,\xi)}\right).$$

This is a smooth irreducible representations of $U(2)$ [15]. Other notation will be explained later.

Theorem I. *Suppose -1 is square in F. Then*

1. *All, but one, smooth characters* (*smooth irreducible representations*) *of* $U(1)$ *occur in the Weil representation* (Theorem 5, Corollary 1, Lemma 20 *and* Theorem 6).

2. *All smooth irreducible representations of $U(2)$, $\rho = \rho\left(\alpha, \varphi, \xi_0\right)$ whose central character, φ, is an extension of some χ_α, where $\alpha = a^2\pi^{-2r-1}, a \in O^\times$, and r is an integer $\geqslant 1$, and ξ_0 is the trivial character of E^1, occur in the Weil representation* (Lemmas 21, 22).

3. *All smooth irreducible representations of $U(2)$, $\rho = \rho(-\alpha, \bar{\varphi}, \bar{\varphi})$ whose central character, φ, is an extension of some χ_α, where $\alpha = a\pi^{-2r-1}$ and a is a non-square element in $O^\times$, and r is an integer $\geqslant 1$, occur in the Weil representation* (Lemmas 23, 24).

4. *Trivial character of $U(2)$ occurs in the Weil representation* (Lemma 29).

5. *None of other representations of $U(2)$ occur in the Weil representation* (Corollary 2, *and this fact that Θ is a bijection*).