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ELSEVIER Linear Algebra and its Applications 401 (2005) 109–117

LINEAR ALGEBRA AND ITS APPLICATIONS

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Spectral refinement on quasi-diagonal matrices

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Received 11 August 2003; accepted 12 December 2003
Available online 21 February 2004
Submitted by J. Queiró

Dedicated to Graciano de Oliveira

Abstract

In several applications needing the numerical computation of eigenvalues and eigenvectors we deal with strongly quasi-diagonal matrices. An iterative explicit method for this kind of problem is proposed here. Its convergence is proved by means of an argument based on the perturbed fixed slope method. Numerical experiments complete this work.

AMS classification: 15A18; 65F15; 47A75

Keywords: Eigenvalues; Eigenvectors; Quasi-diagonal matrices; Perturbed fixed slope; Spectral refinement; Perturbation theory

1. Introduction

Let us consider the problem of computing the eigenpair of a quasi-diagonal matrix. Let $\mathbb{C}^{n\times 1}$ denote the complex linear space with the Euclidean norm $\|\cdot\|$. We recall that the corresponding subordinated operator norm satisfies

$$\|M\| = \sqrt{\rho(M^*M)} \quad \text{for all } M \in \mathbb{C}^{n\times n},$$

where ρ denotes the spectral radius.

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doi:10.1016/j.laa.2003.12.004

We shall deal with a nonsingular matrix $A \in \mathbb{C}^{n\times n}$ and its decomposition

$$A = D + \Delta, \quad \text{where } D(i, j) := \begin{cases} A(i, i) & \text{if } i = j, \\ 0 & \text{if } i \neq j. \end{cases} \tag{1}$$

Let $\delta \geqslant 0$ be given by

$$\delta := \|\Delta\|.$$

We suppose that δ is "small enough" in a sense to be made precise later.

In these circumstances we are interested in computing the eigenvalues and eigenvectors of A iteratively and taking as initial approximations the diagonal entries and the canonical vectors respectively.

2. The problem and the method

We consider here the case of distinct diagonal coefficients:

$$i \neq j \quad \Rightarrow \quad D(i, i) \neq D(j, j). \tag{2}$$

For each $s \in \mathbb{N}$, $1 \leqslant s \leqslant n$, we consider $D(s, s)$ as an approximate eigenvalue of A and the canonical column e_s defined by

$$e_s(i) := \begin{cases} 1 & \text{if } i = s, \\ 0 & \text{otherwise,} \end{cases}$$

as a corresponding approximate eigenvector.

From perturbation theory (see, for instance, [4]) we know that, for each fixed s and for δ small enough, A has an eigenvector x_∞ such that $e_s^* x_\infty = 1$. Thus we may formulate the problem as

$$\text{Find } x_\infty \in \mathbb{C}^{n\times 1} \text{ such that } Ax_\infty = \lambda_\infty x_\infty,\ e_s^* x_\infty = 1. \tag{3}$$

In order to produce an iterative scheme for approximating x_∞ starting from $x_0 := e_s$, we formalize problem (3) as the one consisting of finding the root of some nonlinear differentiable operator with nonsingular first Fréchet derivative and apply to this problem a perturbed fixed slope or modified Newton algorithm. In this context, let us define the operator

$$F := \mathbb{C}^{n\times 1} \to \mathbb{C}^{n\times 1}, \quad F(x) := Ax - xe_s^* Ax. \tag{4}$$

It is evident that any nonzero $x \in \mathbb{C}^{n\times 1}$ satisfying

$$F(x) = 0 \tag{5}$$

is an eigenvector of A corresponding to the eigenvalue

$$\lambda := e_s^* Ax.$$

Moreover, since A is nonsingular, $\lambda \neq 0$ and hence necessarily

$$e_s^* x = 1. \tag{6}$$