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# Values of minors of some infinite families of matrices constructed from supplementary difference sets and their application to the growth problem

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**Abstract**

We obtain explicit formulae for the values of the $2v - j$ minors, $j = 0, 1, 2$ of some infinite families of matrices of order $2v$ constructed from supplementary difference sets of the form $2 - \{v; k_1, k_2; \lambda\}$. This allows us to obtain information on the growth problem for families of matrices with moderate growth. Several examples specifying the pivot pattern of these matrices are presented and a conjecture about their pivot structure is posed.


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## 1. Introduction

Let $A = [a_{ij}] \in \mathscr{R}^{n\times n}$. We reduce $A$ to upper triangular form by using Gaussian elimination with complete pivoting (GECP) [14]. Let $A^{(k)} = [a_{ij}^{(k)}]$ denote the matrix obtained after the first $k$ pivoting operations, so $A^{(n-1)}$ is the final upper triangular matrix. The diagonal entries of that final matrix are called *pivots* and are denoted by $p_i$, $i = 1, 2, \ldots, n$. Matrices with the property that no exchanges are actually needed during GECP are called *completely pivoted* (CP). The following problem arises during the elimination process.

### The growth problem

Let $g(n, A) = \max_{i,j,k} |a_{ij}^{(k)}|/|a_{11}^{(0)}|$ denote the growth factor associated with GECP on $A$ and $g(n) = \sup\{ g(n, A)/A \in \mathscr{R}^{n\times n} \}$. The problem of determining $g(n)$ for various values of $n$ is called the growth problem.

### The importance of the growth factor

For the solution of the linear system $A \cdot \underline{x} = \underline{b}$ the most popular numerical method used for its solution is Gaussian elimination with pivoting. The following theorem holds for the computed solution.

**Theorem 1** [14]. *Let $A \cdot \underline{x} = \underline{b}$ be an m by n system, and let $\delta A$ be an m by n matrix. The computed solution $\underline{\bar{x}}$ using Gaussian elimination with pivoting is the exact solution of the system $(A + \delta A) \cdot \underline{\bar{x}} = \underline{b}$, where*

$$\|\delta A\|_\infty \leqslant (n^3 + 2n^2 + 2n) \cdot g(n) \cdot u_1 \|A\|_\infty,$$

$u_1$ denotes the unit roundoff error. Thus the stability of the computed solution heavily relies on the value of the growth factor $g(n)$. The question naturally arises is: How large can the growth factor $g(n)$ be for an arbitrary matrix $A$ of order $n$? When Gaussian elimination with partial pivoting is used it can be proved [14] that $g(n) \leqslant 2^n$ whereas when Gaussian elimination with complete pivoting is used it can be proved [14] that $g(n) \leqslant \left[n \cdot 2 \cdot 3^{\frac{1}{2}} \cdots n^{\frac{1}{n-1}}\right]^{\frac{1}{2}}$. Both these bounds do not provide stability when they are replaced in the error formula of Theorem 1 and are not realistic since in practice matrices with large growth factor rarely appear. In [1] Cryer did numerical experiments in which he computed $g(n, A)$, doing complete pivoting on $n \times n$ matrices $A$ with entries chosen randomly from the interval $[1, -1]$ and for sizes up to $n = 8$. He had to generate over 50,000 $3 \times 3$ examples before finding one with $g(3, A) > 2$. Also the largest $g(n, A)$ he obtained by testing 10,000 random matrices for sizes up to $n = 8$ was 2.8348. We also performed experiments with random matrices from $[1, -1]$ and the largest $g(n, A)$ we encountered for GECP for size $n = 1000$ was 7.010859. The theoretical bound for this value of $n$ is 8652740.061.