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Values of minors of some infinite families of matrices constructed from supplementary difference sets and their application to the growth problem

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Abstract

We obtain explicit formulae for the values of the 2v - j minors, j = 0, 1, 2 of some infinite families of matrices of order 2v constructed from supplementary difference sets of the form $2 - \{v; k_1, k_2; \lambda\}$. This allows us to obtain information on the growth problem for families of matrices with moderate growth. Several examples specifying the pivot pattern of these matrices are presented and a conjecture about their pivot structure is posed. © 2005 Elsevier Inc. All rights reserved.

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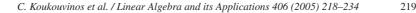
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1. Introduction

Let $A = [a_{ij}] \in \mathscr{R}^{n \times n}$. We reduce A to upper triangular form by using Gaussian elimination with complete pivoting (GECP) [14]. Let $A^{(k)} = [a_{ij}^{(k)}]$ denote the matrix obtained after the first k pivoting operations, so $A^{(n-1)}$ is the final upper triangular matrix. The diagonal entries of that final matrix are called *pivots* and are denoted by p_i , i = 1, 2, ..., n. Matrices with the property that no exchanges are actually needed during GECP are called *completely pivoted* (CP). The following problem arises during the elimination process.

The growth problem

Let $g(n, A) = \max_{i,j,k} |a_{ij}^{(k)}|/|a_{11}^{(0)}|$ denote the growth factor associated with GECP on A and $g(n) = \sup\{g(n, A)/A \in \mathbb{R}^{n \times n}\}$. The problem of determining g(n) for various values of n is called the growth problem.

The importance of the growth factor

For the solution of the linear system $A \cdot \underline{x} = \underline{b}$ the most popular numerical method used for its solution is Gaussian elimination with pivoting. The following theorem holds for the computed solution.

Theorem 1 [14]. Let $A \cdot \underline{x} = \underline{b}$ be an *m* by *n* system, and let δA be an *m* by *n* matrix. The computed solution \underline{x} using Gaussian elimination with pivoting is the exact solution of the system $(A + \delta A) \cdot \underline{x} = \underline{b}$, where

 $\|\delta A\|_{\infty} \leq (n^3 + 2n^2 + 2n) \cdot g(n) \cdot u_1 \|A\|_{\infty},$

 u_1 denotes the unit roundoff error. Thus the stability of the computed solution heavily relies on the value of the growth factor g(n). The question naturally arises is: How large can the growth factor g(n) be for an arbitrary matrix A of order n? When Gaussian elimination with partial pivoting is used it can be proved [14] that $g(n) \leq 2^n$ whereas when Gaussian elimination with complete pivoting is used it can be proved [14] that $g(n) \leq 2^n$ whereas when Gaussian elimination with complete pivoting is used it can be proved [14] that $g(n) \leq [n \cdot 2 \cdot 3^{\frac{1}{2}} \cdots n^{\frac{1}{n-1}}]^{\frac{1}{2}}$. Both these bounds do not provide stability when they are replaced in the error formula of Theorem 1 and are not realistic since in practice matrices with large growth factor rarely appear. In [1] Cryer did numerical experiments in which he computed g(n, A), doing complete pivoting on $n \times n$ matrices A with entries chosen randomly from the interval [1, -1] and for sizes up to n = 8. He had to generate over 50,000 3 \times 3 examples before finding one with g(3, A) > 2. Also the largest g(n, A) he obtained by testing 10,000 random matrices for sizes up to n = 8 was 2.8348. We also performed experiments with random matrices from [1, -1] and the largest g(n, A) we encountered for GECP for size n = 1000 was 7.010859. The theoretical bound for this value of n is 8652740.061.

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