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Stability of polytopes of matrices via affine parameter-dependent Lyapunov functions: Asymptotically exact LMI conditions ☆

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Abstract

This paper investigates necessary and sufficient conditions for the existence of an affine parameter-dependent Lyapunov function assuring the Hurwitz (or Schur) stability of a polytope of matrices. A systematic procedure for constructing a family of linear matrix inequalities conditions of increasing precision is given. At each step, a set of linear matrix inequalities provides sufficient conditions for the existence of the affine parameter-dependent Lyapunov function. Necessity is asymptotically attained through a relaxation based on a generalization of Pólya's Theorem to the case of matrix valued functions. Numerical experiments illustrate the results.

AMS classification: 93D05; 93D09; 15A39; 93C05; 70G55; 52B99

Keywords: Hurwitz stability; Schur stability; Affine parameter-dependent Lyapunov function; Linear matrix inequalities; Polytopes of matrices

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1. Introduction

The stability of compact sets of matrices is a subject that has been studied since many years and presents a wide range of applications in areas such as linear algebra and control systems. Special attention has been devoted to the investigation of Hurwitz and Schur stability of polytopes of matrices using the Lyapunov approach, which has provided appealing numerical tools and can be extended to deal with a great variety of similar problems. Through this approach, the stability of a matrix $A \in \mathbb{R}^{n\times n}$ can be inferred by means of a Lyapunov function $v(x) : \mathbb{R}^n \to \mathbb{R}$ (usually quadratic on x) such that $v(x) > 0$ and $\dot{v}(x) < 0$ for all $x \neq 0$, $x \in \mathbb{R}^n$.

The use of Lyapunov functions to assess the stability of a polytope of matrices gives rise to conditions that can be cast as linear matrix inequalities (LMIs). In the LMI framework, the stability can be verified through convex optimization problems which can be efficiently solved by polynomial time algorithms [1,2]. The quadratic stability, i.e. the use of a quadratic Lyapunov function $v(x) = x'Px$ with a constant symmetric positive definite matrix P not depending on the uncertainty, provides a simple sufficient condition in terms of a set of LMIs described only at the vertices of the polytope. The existence of a feasible solution assures the stability of the entire domain. In control systems, quadratic stability has been used as the starting point to deal with several problems such as robust control and filter design including $\mathscr{H}_2$ and $\mathscr{H}_\infty$ criteria (see [3] and references therein). The results based on quadratic stability are specially adequate to handle time-varying uncertain parameters, but can be very conservative in the evaluation of the stability of polytopes of matrices.

Affine parameter-dependent Lyapunov functions $v(x) = x'P(\alpha)x$, where $P(\alpha)$ depends affinely on α, have been used to reduce the conservatism in the evaluation of stability of matrix polytopes. In [4], the feasibility of a set of LMIs with extra matrix variables provides a set of Lyapunov matrices whose convex combination assures the Hurwitz stability of the polytope. The Schur counterpart results appeared in [5] and the generalization to cope with stability of the polytope of matrices with respect to any convex region in the complex plane in [6]. Exploiting the algebraic structure of the Lyapunov condition applied to a general matrix inside the polytope, sufficient conditions for the existence of an affine parameter-dependent Lyapunov matrix assuring the stability of the polytope have been given in [7] (Schur case) and [8] (Hurwitz case). The conditions in [7,8] are, in general, less conservative than the ones in [4–6], and all of them encompass the quadratic stability as a special case. A numerical comparison can be found in [9] and a more general result (containing the above ones) has been given in [10], where the ideas of introducing extra variables and of exploring the algebraic structure were combined to provide sufficient conditions for the stability of a polytope of matrices with respect to a generic convex region in the complex plane.

Recently, higher degree parameter-dependent Lyapunov functions have been used to provide less conservative evaluations of stability domains. Sufficient LMI condi-