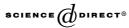


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LINEAR ALGEBRA AND ITS APPLICATIONS

Linear Algebra and its Applications 399 (2005) 203–224

www.elsevier.com/locate/laa

Schur complements and state space realizations[☆]

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> Received 26 April 2004; accepted 21 October 2004 Available online 19 December 2004 Submitted by F. Zhang

Abstract

Motivated by state space realizations of transfer functions from system theory, a number of operations on Schur complements are introduced and studied. These operations are equivalence, extension, multiplication, inversion, and factorization. Together they form an algebraic framework which is of independent interest, and also useful in solving problems in analysis. © 2004 Elsevier Inc. All rights reserved.

AMS classification: primary 47A64, 47A56, 47A68; secondary 47A20, 15A23

Keywords: Equivalence; Extension; Factorization; Realization; Schur complement; Transfer function

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^{*} Work supported in part by the European Community's Human Potential Programme under contract HPRN-CT-2000-00116 [Analysis and Operators].

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0. Introduction

Throughout this paper S will be a (2×2) operator (matrix) of the type

$$S = \begin{bmatrix} A & B \\ C & D \end{bmatrix} : X \dot{+} Y \to \widehat{X} \dot{+} \widehat{Y}, \tag{1}$$

where X, Y, \widehat{X} and \widehat{Y} are complex Banach spaces. Suppose $A: X \to \widehat{X}$ is invertible. Then, by (block) Gauss elimination,

$$S = \begin{bmatrix} I_{\widehat{X}} & 0 \\ CA^{-1} & I_{\widehat{Y}} \end{bmatrix} \begin{bmatrix} A & 0 \\ 0 & D - CA^{-1}B \end{bmatrix} \begin{bmatrix} I_X & A^{-1}B \\ 0 & I_Y \end{bmatrix}, \tag{2}$$

where the first factor and the last factor in the right hand side are both invertible. The second term in the diagonal of the factor in the middle of (2),

$$W_1(S) = W_1\left(\begin{bmatrix} A & B \\ C & D \end{bmatrix}\right) = D - CA^{-1}B : Y \to \widehat{Y}$$

is called the *first Schur complement in S*. Other names that are in vogue are *Schur complement of A in S* and *Schur complement of S relative to A*; also instead of $W_1(S)$, one finds the notation S/A (cf., [20, Chapter 1]).

Analogously, whenever $D: Y \to \widehat{Y}$ is invertible,

$$S = \begin{bmatrix} I_{\widehat{X}} & BD^{-1} \\ 0 & I_{\widehat{Y}} \end{bmatrix} \begin{bmatrix} A - BD^{-1}C & 0 \\ 0 & D \end{bmatrix} \begin{bmatrix} I_X & 0 \\ D^{-1}C & I_Y \end{bmatrix}, \tag{3}$$

and the operator

$$W_2(S) = W_2\left(\begin{bmatrix} A & B \\ C & D \end{bmatrix}\right) = A - BD^{-1}C : X \to \widehat{X}$$

is said to be the second Schur complement in S. Clearly, in this situation,

$$W_2\left(\begin{bmatrix} A & B \\ C & D \end{bmatrix}\right) = W_1\left(\begin{bmatrix} D & C \\ B & A \end{bmatrix}\right),\tag{4}$$

and hence every result for first Schur complements has a counterpart for second Schur complements and vice versa.

Schur complements arise naturally in mathematical system theory. Indeed, when in (1) the spaces X and \widehat{X} coincide and the operator A is replaced with $A - \lambda$ (shorthand for $A - \lambda I_X$), one has

$$W_1\left(\begin{bmatrix} A - \lambda & B \\ C & D \end{bmatrix}\right) = D + C(\lambda - A)^{-1}B. \tag{5}$$

The right hand side of this expression is a state space realization of the transfer function of the linear time invariant system

$$\begin{cases} x'(t) = Ax(t) + Bu(t), & t \ge 0, \\ y(t) = Cx(t) + Du(t), & t \ge 0, \\ x(0) = 0. \end{cases}$$
 (6)

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