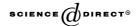


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On similarity of matrices over commutative rings

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Abstract

The paper studies the problem on matrix similarity over a commutative rings. The conditions are determined, under which the matrix is similar to a companion or diagonal matrices. © 2004 Elsevier Inc. All rights reserved.

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Let R be a commutative ring with the unit element $e \neq 0$ and U(R) the set of divisors of unit element e. Further, let $R_{n,m}$ and $R_{n,m}[x]$ be the sets of $n \times m$ matrices over R and the polynomial ring R[x], respectively. Denote by I and O, respectively, the identity and zero matrices of order n. Let $b(x) = x^k - b_1 x^{k-1} - b_2 x^{k-2} - \cdots - b_{k-1} x - b_k$ be a monic polynomial of degree k in R[x]. The $k \times k$ matrix

$$F_b = \begin{bmatrix} 0 & e & 0 & \cdots & 0 \\ 0 & 0 & e & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & e \\ b_k & b_{k-1} & \cdots & b_2 & b_1 \end{bmatrix}$$

is called the companion matrix of b(x).

Recall two matrices A and B in $R_{n,n}$ are said to be similar, if $A = TBT^{-1}$ for some matrix $T \in GL(n, R)$. Let $A \in R_{n,n}$ be a matrix with a characteristic polynomial

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$$\det(Ix - A) = a(x) = x^{n} - a_{1}x^{n-1} - a_{2}x^{n-2} - \dots - a_{n-1}x - a_{n}.$$

The aim of the present paper is to find the conditions of similarity of matrices A and F_a . In case, if $\det(Ix - A) = \prod_{k=1}^n (x - \alpha_k)$, we establish conditions for existence of matrix $T \in GL(n, R)$ such that $TAT^{-1} = \operatorname{diag}(\alpha_1, \alpha_2, \dots, \alpha_n) \in R_{n,n}$ —a diagonal matrix. For the case, where R is a field or a ring, these problems were studied in [1–7].

Proposition 1. Let the characteristic polynomial of a matrix $A \in R_{n,n}$ admit the representation

$$\det(Ix - A) = d_1(x)d_2(x)\dots d_m(x),$$

where $d_i(x) = x^{k_i} - d_{i,1}x^{k_i-1} - \cdots - d_{i,k_i-1}x - d_{i,k_i}$ – are monic polynomials over R, $\deg d_i(x) = k_i$, $1 \le k_i \le n$, $i = 1, 2, \ldots, m$; $k_1 + k_2 + \cdots + k_m = n$. If the matrix A is similar to a block diagonal matrix $D_A = \operatorname{diag}(F_{d_1}, F_{d_2}, \ldots, F_{d_m})$, then for every matrix $T \in GL(n, R)$ such that $TAT^{-1} = D_A$, there exist rows $t_1, t_2, \ldots, t_m \in R_{1,n}$ such that

$$T = \begin{pmatrix} T_1 \\ T_2 \\ \vdots \\ T_m \end{pmatrix}, \quad \text{where } T_i = \begin{pmatrix} t_i \\ t_i A \\ \vdots \\ t_i A^{k_i - 2} \\ t_i A^{k_i - 1} \end{pmatrix}, \ i = 1, 2, \dots, m.$$

Proof. Let $T \in GL(n, R)$ be such matrix that

$$TAT^{-1} = D_A = \text{diag}(F_{d_1}, F_{d_2}, \dots, F_{d_m}).$$
 (1)

Note that T can be rewritten as

$$T = \begin{pmatrix} T_1 \\ \vdots \\ T_i \\ \vdots \\ T_m \end{pmatrix}, \text{ where } T_i = \begin{pmatrix} \bar{t}_{i,1} \\ \bar{t}_{i,2} \\ \vdots \\ \bar{t}_{i,k_i-1} \\ \bar{t}_{i,k_i} \end{pmatrix} \in R_{k_i,n}; \ \bar{t}_{i,j} \in R_{1,n};$$

 $j=1,2,\ldots,k_i;\ i=1,2,\ldots,m$. From equality (1) we find that $T_iA=\|O_1 \quad F_{d_i} \quad O_2\|T$, where O_1 and O_2 are zero $(k_i\times (k_1+\cdots+k_{i-1}))$ and $(k_i\times (k_{i+1}+\cdots+k_m))$ matrices respectively. From the last equality we conclude that

$$\begin{cases} \bar{t}_{i,1}A = \bar{t}_{i,2}, \\ \bar{t}_{i,2}A = \bar{t}_{i,1}A^2 = \bar{t}_{i,3}, \\ \dots & \dots \\ \bar{t}_{i,k_i-2}A = \bar{t}_{i,1}A^{k_i-2} = \bar{t}_{i,k_i-1}, \\ \bar{t}_{i,k_i-1}A = \bar{t}_{i,1}A^{k_i-1} = \bar{t}_{i,k_i}. \end{cases}$$

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