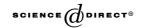


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LINEAR ALGEBRA AND ITS APPLICATIONS

Linear Algebra and its Applications 397 (2005) 99-106

www.elsevier.com/locate/laa

Tridiagonal pairs and the Askey–Wilson relations

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Received 16 August 2004; accepted 7 October 2004

Submitted by R.A. Brualdi

Abstract

The notion of a tridiagonal pair was introduced by Ito, Tanabe and Terwilliger. Let V denote a nonzero finite dimensional vector space over a field \mathscr{F} . A tridiagonal pair on V is a pair (A, A^*) , where $A: V \to V$ and $A^*: V \to V$ are linear transformations that satisfy some conditions. Assume (A, A^*) is a tridiagonal pair on V. Recently Terwilliger and Vidunas showed that if A is multiplicity-free on V, then (A, A^*) satisfy the following "Askey–Wilson relation" for some scalars $\beta, \gamma, \gamma^*, \varrho, \varrho^*, \omega, \eta, \eta^*$.

$$A^{2}A^{*} - \beta AA^{*}A + A^{*}A^{2} - \gamma (AA^{*} + A^{*}A) - \varrho A^{*} = \gamma^{*}A^{2} + \omega A + \eta I,$$

$$A^{*^{2}}A - \beta A^{*}AA^{*} + AA^{*^{2}} - \gamma^{*}(A^{*}A + AA^{*}) - \varrho^{*}A = \gamma A^{*^{2}} + \omega A^{*} + \eta^{*}I.$$

In the paper, we show that, if a tridiagonal pair (A, A^*) satisfy the Askey–Wilson relations, then the eigenspaces of A and the eigenspaces of A^* have one common dimension, and moreover if \mathscr{F} is algebraically closed then that common dimension is 1. © 2004 Elsevier Inc. All rights reserved.

AMS classification: 05E30; 05E35; 33C45; 33D45

Keywords: Askey-Wilson relation; Tridiagonal pair; Leonard pair

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1. Introduction

The notion of a tridiagonal pair was introduced by Ito, Tanabe and Terwilliger, that is defined as follows.

Definition 1.1 [1]. Let V denote a nonzero finite dimensional vector space over a field \mathscr{F} . By a tridiagonal pair on V, we mean a pair (A, A^*) , where $A: V \longrightarrow V$ and $A^*: V \longrightarrow V$ are linear transformations that satisfy the following conditions.

- (i) A and A^* are both diagonalizable on V.
- (ii) There exists an ordering V_0, V_1, \ldots, V_d of the eigenspaces of A such that

$$A^*V_i \subseteq V_{i-1} + V_i + V_{i+1} \quad (0 \le i \le d),$$

where
$$V_{-1} = 0$$
, $V_{d+1} = 0$.

(iii) There exists an ordering $V_0^*, V_1^*, \dots, V_\delta^*$ of the eigenspaces of A^* such that

$$AV_i^* \subseteq V_{i-1}^* + V_i^* + V_{i+1}^* \quad (0 \le i \le \delta),$$

where
$$V_{-1}^* = 0$$
, $V_{s+1}^* = 0$.

where $V_{-1}^*=0$, $V_{\delta+1}^*=0$. (iv) There is no subspace W of V such that both $AW\subseteq W$, $A^*W\subseteq W$, other than W = 0 and W = V.

Remark 1.2. With reference to Definition 1.1, it is known that $d = \delta$ and dim $V_i =$ dim V_i^* (0 $\leq i \leq d$) [1–Lemma 4.5 and Corollary 5.7].

Throughout this paper, let (A, A^*) denote a tridiagonal pair on a vector space Vover a field \mathcal{F} .

The notion of a Leonard pair was introduced by Terwilliger [3]. A Leonard pair (A, A^*) coincides with a tridiagonal pair such that each eigenspace of A is one dimensional. Recently Terwilliger and Vidunas [4-Theorem 1.5] showed that every Leonard pair (A, A^*) satisfies the following relations for some scalars $\beta, \gamma, \gamma^*, \varrho$, $\varrho^*, \omega, \eta, \eta^*.$

$$A^{2}A^{*} - \beta AA^{*}A + A^{*}A^{2} - \gamma (AA^{*} + A^{*}A) - \varrho A^{*} = \gamma^{*}A^{2} + \omega A + \eta I,$$

$$A^{*2}A - \beta A^{*}AA^{*} + AA^{*2} - \gamma^{*}(A^{*}A + AA^{*}) - \varrho^{*}A = \gamma A^{*2} + \omega A^{*} + \eta^{*}I.$$
(2)

These relations are the Askey-Wilson relations, which was introduced by Zhedanov

In the present paper, we prove the following results.

Theorem 1.3. Suppose

$$A^{2}A^{*} - \beta AA^{*}A + A^{*}A^{2} - \gamma (AA^{*} + A^{*}A) - \varrho A^{*} = \varrho(A)$$
(3)

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