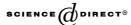


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## Siegel transformations for even characteristic

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#### Abstract

Let V be a vector space over a field K of even characteristic and |K| > 3. Suppose K is perfect and  $\pi$  is an element in the special orthogonal group SO(V) with dim  $B(\pi) = 2d$ . Then  $\pi = \rho_1 \cdots \rho_{d-1} \kappa$ , where  $\rho_j$ ,  $j = 1, \ldots, d-1$ , are Siegel transformations and  $\kappa \in SO(V)$  with dim  $B(\kappa) = 2$ . The length of  $\pi$  with respect to the Siegel transformations is d if  $\pi$  is unipotent or if dim  $B(\pi)/\text{rad }B(\pi)\geqslant 4$ ; otherwise it is d+1. © 2004 Elsevier Inc. All rights reserved.

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#### 1. Introduction

Let G be a group and let S be a set of generators for G. Let  $\pi$  be an element in G, then  $\pi = s_1 \cdots s_k$ , where all  $s_j$ ,  $j = 1, \ldots, k$ , are elements in S. The *length*  $\ell(\pi)$  of  $\pi$  with respect to S is the minimal k for which such a factorization exists. For certain groups G and certain generating systems S it is possible to determine  $\ell(\pi)$  for each  $\pi$  in G.

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Bachmann [1] coined the phrase *length problem* for the program described above. The length problem for the orthogonal groups over fields of characteristic not 2 was solved by Scherk [12]. Further, Dieudonné [3] solved the length problem for several other classical groups. For more references see e.g. Ellers [4,5].

If Q is a nondegenerate singular quadratic form on a vector space V over a field K, then the commutator subgroup  $G = \Omega(V)$  of O(V) is generated by the set of Siegel transformations. Assuming that the field K of coefficients has characteristic not 2, Knüppel solved the length problem for  $G = \Omega(V)$ , where the generating set S is the set of Siegel transformations [10]. In the present paper, we assume that the characteristic of K is even, |K| > 3, and K is perfect. Under these conditions we solve the length problem for  $\Omega(V)$  with respect to Siegel transformations.

In Section 3, we are laying the groundwork. Here we assume that V is nonsingular and that V contains singular vectors distinct from zero. We see that some of the properties established in [10] for orthogonal groups over fields K of characteristic not 2 are also valid when the characteristic of K is even.

In Section 4, we establish a lower bound for the Siegel length of an isometry. In Section 5, we assume that K is perfect, and we determine the Siegel length of an isometry  $\pi$ , Theorem 5.5. Here our approach differs entirely from that in [10]. Our tools include the factorization of an orthogonal transformation  $\pi$  into a product of two involutions [6,7] and also the factorization of  $\pi$  into a product  $\pi = \mu \cdot \nu$ , where  $\mu$  is unipotent and the path of  $\nu$  is nonsingular [8].

#### 2. Notation

Let V be a vector space of dimension n over a field K where |K| > 2, equipped with a quadratic form Q (see  $[2-\S16]$ ), defined by  $Q(\alpha v) = \alpha^2 Q(v)$  and Q(v+w) = Q(v) + Q(w) + f(v,w) for some bilinear form f, where  $\alpha \in K$  and  $v,w \in V$ . Two vectors  $v,w \in V$  are called perpendicular,  $v\perp w$ , if f(v,w) = 0. A vector  $v\in V$  is called isotropic if f(v,v) = 0 and singular if Q(v) = 0. Let W be a subspace of V. Then W is called totally isotropic if f(u,w) = 0 for all  $u,w \in W$  and totally singular if Q(w) = 0 for all  $w \in W$ . A totally singular subspace is also totally isotropic, but the converse is not necessarily true. The subspaces rad  $W = W \cap W^{\perp}$  and  $SW = \{x \in \text{rad } W \mid Q(x) = 0\}$  are called the radical of W and the singular of W, respectively. The space W is said to be nonsingular if rad W = 0.

The orthogonal group on V, denoted O(V), is the set of isometries, i.e. of all transformations that preserve the value of Q. For  $\pi \in O(V)$  we define  $B(\pi) := V(\pi-1)$  and  $F(\pi) := \ker(\pi-1)$ . The subspaces  $B(\pi)$  and  $F(\pi)$  of V are called P and P and P and P are called P and P and P are called P are called P and P are called P and P are called P are called P are called P and P are called P and P are called P are called P are called P and P are called P and P are called P

We shall always assume that V is nonsingular and that there is at least one  $v \in V \setminus \{0\}$  such that Q(v) = 0.

We shall state a number of facts (see e.g. [13]).

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